The Variability of Velocity of Money in a Search Model*

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Abstract

We construct a dynamic equilibrium model where there is costly search in the goods market and the labor market. Incorporating shocks to money growth and productivity, we calibrate the model to the US time series data to examine the model's quantitative predictions on aggregate variables and, in particular, on the variability of consumption velocity of money. Despite the fact that money is the only asset, the model captures most of the variability of velocity in the data. It also generates realistic predictions on the moments of other variables and provides peresistent propagation of the shocks. The model generates these results largely because costly search gives an important role to the extensive margin of trade.

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1. Introduction

In this paper we construct a stochastic monetary model where there is costly search in both the goods market and the labor market. After calibrating the model to the US data, we compare the statistical moments of aggregate variables in the model with the sample moments. In particular, we focus on the short-run (quarterly) variability of consumption velocity of money.

Figures 1 and 2 here.

Velocity varies considerably in the US data, both in the long run and in the short run. Figure 1 depicts the raw series and the filtered series of consumption velocity of money in the period 1959:I – 1998:III, using $M1$ as the monetary aggregate and the expenditure on non-durable goods as consumption.[1] Figure 2 depicts velocity of $M2$. Three features are noticeable from these figures. First, $M1$ velocity has a growing trend, with a brief interruption in the 1980s. Second, $M2$ velocity does not seem to have a growing trend, but there was a distinct jump in the 1990s. Third, both $M1$ velocity and $M2$ velocity are volatile, as illustrated by the filtered series. It is not difficult to explain the first two features. At the annual frequency, the trend and large shifts in velocity can be explained well by changes in interest rates (e.g., Lucas, 1988, and McGrattan, 1998) and by shifts in expectations about government policy (e.g., Gordon et al., 1998). However, these explanations fail to account for the large variability of velocity at the frequency of business cycles in the filtered data.

In fact, previous attempts to use general equilibrium models to explain the variability of velocity have had very limited success. For example, in the simplest model where all consumption is purchased with cash in advance, velocity is constant at unity. In an attempt to overcome this difficulty, Hodrick et al. (1991) endogenize velocity by introducing credit goods and different information structures. Despite these modifications, the variability of velocity is less than 40% of that in the data (Table 6 therein) and the moments of some key variables are unrealistic. This low variability of velocity remains a feature of general equilibrium models even when the liquidity

[1] Throughout this paper, the filtering procedure is the filter used by Hodrick and Prescott (1980).

effect is introduced to increase the volatility of interest rates (see Christiano, 1991). In this paper, we investigate whether alternative models can account for the variability of velocity.

There are two motivations for focusing on the velocity of money. One is that understanding short-run fluctuations of velocity is important for understanding the role of money in business cycles. According to the monetarists, changes in the money stock are important sources of output fluctuations. Central to this view is the assumption that velocity can be expressed as a stable function of a few macro variables, such as interest rates.[2] A large variability of velocity at the business cycle frequency presents a challenge to this assumption, especially when most of it cannot be explained by variations in the macro variables. Even if one is not interested in the monetarists' view, velocity is still a useful indication of how well a model can explain monetary business cycles. For example, the failure of traditional monetary models in accounting for the variability of velocity is often accompanied by their weak propagation of monetary shocks. Thus, finding an explanation for the large variability of velocity may also provide a useful lead to the search in the future for a strong monetary propagation mechanism.

The other motivation for our analysis is to investigate the quantitative predictions of search models. Over the last fifteen years or so, monetary theorists have developed search models from the rudimentary setup of Kiyotaki and Wright (1989, 1993) into a comprehensive microfoundation of money. However, most of these contributions have been theoretical. In this paper, we attempt to show that search models also have interesting quantitative predictions that are different from traditional models. Velocity is a convenient dimension to achieve this purpose. This is so not only because traditional models have had difficulty to generate sufficient variability of velocity, but also because velocity is a more meaningful concept to individual agents in a search model than in traditional models. In traditional models, velocity is a summary statistic that may not have any direct influence on individuals' decisions. In contrast, velocity in a search model captures the precise notion of how many times money has been spent in trade. This notion is tightly related to the frequency of trade which directly influences agents' search decisions.

[2] See Friedman (1956) and Brunner and Meltzer (1963) for the discussion along this line.

The model in this paper has costly search in the goods market and the labor market. Costly search in the goods market, together with particular patterns of preferences and production, makes fiat money a valuable medium of exchange. In contrast to standard search models (e.g., Shi, 1995, and Trejos and Wright, 1995), money is divisible and goods can be stored as inventory by the producers. The aggregate number of matches is an increasing function of buyers' search intensity and sellers' inventory. Costly search in the labor market is modeled in the standard way as in Mortensen (1982) and Pissarides (1990), where the aggregate number of matches is an increasing function of the numbers of vacancies and unemployed workers. In both markets, bargaining determines the terms of trade between the two agents in a match. This economy is exposed to shocks to productivity and money growth.

In this model, velocity is determined by the "extensive margin" of trade. More precisely, consumption velocity of money is equal to the frequency of trades per buyer in the goods market. Because search is costly in the goods market, higher search intensity leads to a higher frequency of trades for a buyer. On the other hand, costly search in the labor market delays the response of employment to shocks and makes the output response sluggish. As a result, the supply of goods and the trading frequency will also depend on sellers' existing inventory. In equilibrium, the frequency of trades per buyer is an increasing function of aggregate search intensity per buyer and aggregate inventory per seller. Shocks change velocity of money by affecting households' decisions on buyers' search intensity and sellers' inventory. This is the mechanism we try to capture with costly search in the two markets.

The mechanism propagates a shock as follows. When a shock is realized, consumption and search intensity respond immediately. These initial responses will change the level of inventory in the next period and, in the presence of sluggish output, they will also change the supply of goods in the next period. The change in the future supply of goods will in turn change search intensity, velocity and consumption in the future. Consider a positive shock to money growth, for example. The shock immediately increases expected inflation, reduces the real money balance and hence reduces consumption. With risk aversion, households try to reduce the fall in consumption by increasing buyers' search intensity. Thus, velocity rises immediately. Moreover, since output is sluggish due to costly labor market search, the fall in current consumption also increases inventory and the supply of goods in the next period. With more goods available, consumption in the next period will rise and households will reduce search intensity. Velocity will fall. These effects in the second period will persist as inventory and the supply of goods will decline only gradually toward the stead state.

We calibrate the model to the US data. To focus on the role of search in explaining velocity, we abstract from assets other than money. In the quantitative exercises, we take $M2$, rather than $M1$, as the aggregate money stock. One reason is that $M2$ velocity is stable in the sample period but $M1$ velocity is not. The other reason is that we want to compare our results with those in Hodrick et al. (1991), who focused on $M2$ velocity. Also for comparability, we incorporate both nominal shocks (to money growth) and real shocks (to productivity). The processes of these shocks are estimated using the vector auto-regression (VAR).

We use the coefficient of variation to measure the variability of a variable, as Hodrick et al. (1991) do. Our model accounts well for the variability of consumption velocity of money. With realistic parameter values, the model is able to capture about 90% of the variability of velocity in the quarterly data. In contrast, Hodrick et al. (1991) can only generate a maximum 40% of the variability of velocity in the data. Even this number was obtained with unrealistic values of the real interest rate.

An obvious question is whether the high variability of velocity in our model is generated by compromising the quantitative performance in other dimensions. The answer is no. To support this answer, we examine the joint distribution of the second moments of other endogenous variables. There are two main findings here. First, the higher variability of velocity in the model is not generated by making output unrealistically volatile. To the contrary, the volatility of output is about 60% of that in the data. Second, the correlations between endogenous variables are realistic and some of them improve upon those obtained by Hodrick et al. (1991). A notable example is the correlation between velocity and consumption growth. This correlation is negative in our model and the magnitude is comparable with that in the data. In contrast, the correlation is either positive or close to zero in Hodrick et al. (1991).

Our findings seem to rely heavily on search in both the goods market and the labor market. If there is no search in the goods market, then velocity will be determined by the intensive margin of trade, as in traditional models of money, and hence it will not be volatile. On the other hand, if there is no search in the labor market, then employment and output will respond to shocks immediately. Since these responses change the supply of goods immediately, inventory plays a much less important role in the supply of goods, and so velocity will be less volatile.

We also find that shocks to money growth and productivity are both important for the model's performance. When there are no shocks to money growth, the model fails to capture some basic monetary features in the data, such as the positive correlation between inflation and the nominal interest rate. On the other hand, when productivity is fixed, the volatility of velocity in the model falls from 90% to 50% of that in the data. Moreover, the response of output to money growth shocks alone is small.

Our model is related to three strands of literature. The first is monetary search models, such as Shi (1995) and Trejos and Wright (1995). We extend these models to allow for divisible money and inventory. Relative to similar extensions carried out before (e.g., Shi, 1997, 1998, and Lagos and Wright, 2002), the new dimensions of the current model are the stochastic components and the quantitative exercises. The second strand of literature is search models of unemployment, e.g., Mortensen (1982) and Pissarides (1990). We follow this literature closely to model labor market search. The third strand of literature empirically estimates the relationship between velocity and other aggregate variables, e.g., Brunner and Meltzer (1963) and Lucas (1988). In contrast to this literature, we compute an equilibrium model, as Hodrick et al. (1991) do. The advantage of an equilibrium model is that it reveals how shocks affect endogenous variables simultaneously.[3]

[3]Gordon and Leeper (2000) and Gordon et al. (1998) also compute general equilibrium models. Their analyses have implications on the behavior of velocity. To create a positive value of money in the equilibrium, they assume a transaction technology in the goods market which entails the use of money. Besides this difference in the model, their focus is different. They examine how agents' expectations of monetary and fiscal policies affect the trend and cyclical features of velocity.

It is useful to clarify that the shocks to money growth in this paper are not necessarily monetary policy shocks. To identify policy shocks, we need to impose proper restrictions on the VAR estimation of the shocks. Such identification is important for analyzing the effects of policy shocks but it is quite subtle (see Christiano et al., 1999). Since the effects of policy shocks are not the focus in this paper, we follow Hodrick et al. (1991) to impose no restriction on the VAR estimation. Nevertheless, our analysis may be a useful precursor to an investigation in the propagation of monetary policy shocks, as we will discuss further in Section 7.

2. The Description of the Economy

2.1. The Household and Matches

The model economy has discrete time and many types of households. The number of households in each type is large and normalized to one. A household of each type produces a specific good which the household does not consume but which is desired by some other types of households. Households meet with each other bilaterally according to a matching function described later. The focus of this analysis is on the matches with a single coincidence of wants, which we call *trade matches*. In these matches, fiat money is used as a medium of exchange. Non-monetary trades are not explicitly modeled here. However, to facilitate the calibration later, we summarize all non-monetary trades as providing an amount of consumption that is exogenous to the household.[4] Similarly, we abstract from all assets other than money.

The model has two elements that are not standard in search models. One is labor market search and the other is inventory. As discussed in the introduction, both elements are important for the volatility of velocity. To allow for inventory, we assume that goods can be stored by, and only by, the producers. The presence of inventory does not destroy the essential role of money. Because goods can only be stored by the producers, they cannot circulate as a means of payment. Also, claims on inventory will not be valued in equilibrium, because a holder of a claim has zero probability of meeting the issuer and redeeming the claims.

[4] For search models that explicitly model barter, see Kiyotaki and Wright (1991, 1993), Shi (1995, 1997) and Trejos and Wright (1995).

Random matching in the goods market can induce a non-degenerate distribution of money holdings across agents. Keeping track of this distribution is analytically intractable and computationally difficult. To maintain tractability, we extend the household structure in Lucas (1990) to allow each household to perfectly smooth the matching risks. More precisely, each household consists of a large number of members who regard the household's utility as the common objective and carry out the household's trading decisions. The members in a household do not have incentives and hence do not make decisions; instead, the household makes all the decisions.[5] With this modeling device, the decisions are the same for all households in a symmetric equilibrium, except for the types of goods they consume and produce. Thus, we can examine a representative household's decisions. To the extent that idiosyncratic matching risks can increase consumption volatility, the risk-smoothing assumption under-estimates the variability of velocity.

Let us pick an arbitrary household of an arbitrary type as a "representative" household. Use lower-case letters to denote this household's decisions. Add a hat to other households' decisions and aggregate variables, which the representative household takes as given.

A household has five types of members at any given point of time. These types, and the sizes in parentheses, are as follows: buyers (b), sellers/entrepreneur (n_p), employed workers ($\widehat{n}_w$), unemployed workers (u), and leisure seekers (n_0). The total size of a household's members is 1; i.e., the condition, $b+n_p+\widehat{n}_w+u+n_0=1$, always holds.[6] The hat on the symbol $\widehat{n}_w$ signifies the fact that it is chosen by other households, because the representative household's members who work are employed by other households. For simplicity, we exclude home production from our model. Among the five numbers, $(\widehat{n}_w, n_0)$ are variables but (b, u, n_p) are assumed to be constant, and so any change in the number of employed workers must be accompanied by an opposite

[5] As explained by Shi (1997), this household structure is intended to approximate a single agent's decision problem in the following way. The agent has one unit of time in each period and can divide the time into a large number of bits. Matches take place over these bits of time. At the beginning of each period, the agent makes all the trading decisions in the period and programs the decisions for each bit of time into a separate machine, which corresponds to a member in the household structure. As the period unfolds, the machines carry out the trades according to the programs. At the end of the period, the agent pools the receipts from the machines and consumes. Similar risk-smoothing assumptions are used in labor economics (Rogerson, 1988, and Hansen, 1985). For an alternative set of assumptions that achieves the same purpose, see Lagos and Wright (2002).

[6] One can explore an alternative household structure in which workers are also buyers in the goods market. If the household can choose the intensity with which a worker performs each of the two roles, then this alternative structure will be identical to our model.

change in the number of leisure seekers.[7] Notice that we count entrepreneurs as being employed, and so the total number of employed agents is $(n_p + \widehat{n}_w)$. Define $\widehat{l} = \widehat{n}_w / n_p$ and $B = b/n_p$.

For each buyer, the household chooses the search effort or intensity, e, and the terms of trade to be proposed in each match. For each entrepreneur, the household chooses the number of vacancies, v, and the wage rate to be proposed. The household's utility in a period is

$$U(c) - (\widehat{n}_w + n_p)\varphi - b\Phi(e) - n_p H(v).$$

Here $U(c)$ is the utility of consumption; φ is the disutility of being employed (either as a worker or as an entrepreneur); $\Phi(e)$ is the disutility of a buyer's search intensity; and $H(v)$ is the cost of maintaining a number v of vacancies.[8] Notice that all members consume the same amount as a result of consumption sharing within the household. Assume that $U(.)$ is strictly increasing and concave, that $\Phi(.)$ is increasing and convex, and that $\Phi(0) = \Phi'(0) = 0$. The function $H(.)$ has properties similar to $\Phi(.)$.

In the labor market, an unemployed worker's search effort is assumed to be inelastic.[9] We assume that workers are only matched with other households that do not produce their consumption goods. Thus, wages are paid in terms of money. Let $\widehat{v}$ be the number of vacancies per firm in a period, so that the total number of vacancies is $n_p \widehat{v}$. As is standard in labor search models (see Blanchard and Diamond, 1989), the total number of matches between firms and workers is given by a function $(n_p \widehat{v})^{\psi} u^{1-\psi}$, where $\psi \in (0, 1)$. The matching rate per vacancy is

$$\widehat{\mu} \equiv (n_p \widehat{v}/u)^{\psi - 1}. \tag{2.1}$$

For a firm with v vacancies, the number of matches is $\widehat{\mu} v$. The matching rate per unemployed worker is $(n_p \widehat{v}/u)^{\psi}$. As in a typical search model of unemployment (e.g., Pissarides, 1990), each

[7] The assumption of a constant number of buyers is without loss of generality. Because we will endogenize buyers' search intensity, the effective number of buyers is endogenous. Allowing the household to choose the number of buyers, as well as search intensity, complicates the algebra without adding new results. Similarly, the effective number of sellers is endogenous, as we will describe later.

[8] Alternatively, one can model the vacancy cost in terms of real resources. Then, the main analytical difference will be that a change in the number of vacancies will affect output directly, in addition to the indirect effect through employment. However, this direct effect will be very small in the quantitative exercises, because the vacancy cost is calibrated to be a very small fraction (2%) of the wage bill.

[9] Endogenizing an unemployed worker's search intensity would improve the quantitative performance of the model, but not by much. The reason is that unemployment is very persistent in the data and is much less volatile than vacancy (see Layard et al., 1991).

employed worker separates from the job with an exogenous probability $\delta_w \in (0,1)$ at the end of each period.

We describe the matching function in the goods market similarly. Let $\widehat{e}$ be the search effort per buyer in a period so that $b\widehat{e}$ is the effective number of buyers in the market. Let $\widehat{k}$ be the level of inventory per seller at the beginning of a period and let $s(\widehat{k}_t)$ be a function, explained later, that converts a seller's inventory into the seller's intensity. The effective number of sellers in the market is $n_p s(\widehat{k})$. The total number of trade matches is:

$$\widehat{G} = g_0 (b\widehat{e})^{\xi} [n_p s(\widehat{k})]^{1-\xi}, \qquad \xi \in (0,1), \tag{2.2}$$

where $g_0 > 0$ is a constant.[10] Denote $z = (b\widehat{e})/[n_p s(\widehat{k})]$ as the tightness of the goods market. The matching rate is $\widehat{G}_b$ per unit of buyer's intensity and $\widehat{G}_s$ per unit of seller's intensity, where

$$\widehat{G}_b = g_0 z^{\xi-1}, \quad \widehat{G}_s = g_0 z^{\xi}. \tag{2.3}$$

Thus, a buyer searching with intensity e gets a trade match with probability $e\widehat{G}_b$ and a seller with an inventory k gets a trade match with probability $s(k)\widehat{G}_s$.

The function $s(k)$ requires an explanation. Its appearance in the matching function captures the intuitive idea that the number of trade matches depends on inventory per seller, as well as the number of sellers in the market. This is similar to the idea that the number of matches in the labor market depends on the number of vacancies per firm as well as the number of firms. The function s can be interpreted as the number of shops or warehouses per seller that are stocked up. High inventory reduces the probability of stock-out, and hence increases the probability of trade for the seller. We capture this benefit of inventory to successful trades by assuming $s' > 0$. To rule out corner solutions for inventory, we also assume that this benefit of inventory to the match formation diminishes at the margin, i.e., $s'' < 0$.

[10] The Cobb-Douglas matching function is convenient because it is used in the labor market. Changing the matching function can affect the results primarily by changing the elasticity of the matching rates to the market tightness. This elasticity is captured by ξ in the Cobb-Douglas function. Later in section 6.1, we will find that the quantitative results are not sensitive to ξ. In this sense, our results might also be robust to mild changes in the form of the matching function.

2.2. Aggregate Shocks and the Timing of Events

There are two aggregate shocks in the economy, one to productivity and the other to the money growth rate. Let l be the number of workers per firm, defined as $l_t = n_{wt}/n_p$. Output of a firm is $y_t = f(l_t) = A_t l_t^{\alpha}$, where A is stochastic and $\alpha \in (0, 1)$ is a constant.[11] To describe money growth, let M_t denote the aggregate money stock per household in period t. At the beginning of period $t+1$, a lump-sum transfer $(\gamma_{t+1} - 1)M_t$ is given to each household. Thus, the aggregate money stock grows between t and $t+1$ at a gross rate $\gamma_{t+1} = M_{t+1}/M_t$. We assume that $\ln A$ and γ follow first-order vector auto-regressive processes, which will be described in Section 4.

Because the money stock grows over time, nominal variables are not stationary. To maintain stationarity, we follow the convention (e.g., Lucas, 1990) to normalize nominal variables by the aggregate money stock. We also suppress the time subscript t whenever it is possible and shorten the time subscript $t \pm j$ to $\pm j$ for $j \geq 1$.

The events in a period unfold in the following sequence. At the beginning of the period, the two aggregate shocks are realized and each household receives monetary transfers. The aggregate money stock is measured as M after the shocks are realized. Also measured at this time are the representative household's endogenous state variables: money holdings, m, the level of inventory per firm in the household, k, and the number of employed workers per firm in the household, l. Next, the household chooses the number of vacancies for each firm to maintain, v, and search intensity for each buyer, e. The household also chooses the wage rate w for each firm to offer in a match that was formed in the previous period and chooses the terms of trade for each buyer to offer in a current match in the goods market, (q, x). Here, x is the quantity of money that the buyer offers to the seller and q is the quantity of goods the buyer asks for. After these decisions, the members go to the markets, matches are formed, and the members carry out the trades according to the household's instructions. In the meantime, the workers who were matched two or more periods ago and who have not separated from the firms produce and obtain wages.

[11] This production function implicitly incorporates the entrepreneur's own labor input. For example, we can rewrite the production function as $f(l) = Al^{\alpha}1^{1-\alpha'}$, for some $\alpha' \in (0, 1-\alpha]$, where the entrepreneur's labor input is one unit.

Then, the members go home and pool the receipts from the trades. Every member consumes the same amount, c. Finally, exogenous separation occurs with probability δ_w to matches that just produced (which do not include the matches newly formed in the current period). Also, inventory depreciates at rate δ_k.

Let us clarify a few aspects of the above description. First, to avoid double counting the time that is available to an agent, we assume that a worker who finds a job in a period starts working in the next period rather than the current period. Second, this one-period delay in employment does not create wage rigidity, because we assume that the wage rate for a newly matched worker is negotiated when the worker starts to worker. This explains why the wage offer w in the current period is for the workers who formed matches in the previous period. Third, the variables m, x and w are all normalized by M, and so they are stationary despite money growth.

2.3. The Household's Decision Problem

Before laying out the representative household's decision problem, we describe the household's trading decisions first. Denote the household's value function as $J(m, n_p k, n_p l)$, where the dependence on aggregate variables is suppressed. Denote the expected marginal value of each of the stock variables in the next period, discounted to the current period, as follows:

$$\omega_m = E\left[\frac{\beta}{\gamma_{+1}} \frac{\partial}{\partial m_{+1}} J(m_{+1}, n_p k_{+1}, n_p l_{+1})\right], \tag{2.4}$$

$$\omega_j = E\left[\beta \frac{\partial}{\partial (n_p j_{+1})} J(m_{+1}, n_p k_{+1}, n_p l_{+1})\right], \; j = k, l. \tag{2.5}$$

Here the expectations are conditional on the information available in the current period after the shocks are realized. Notice that the future value of money is discounted by the money growth rate γ_{+1}, as well as by β, because m is a variable normalized by the aggregate money stock.

Consider a match in the labor market that was formed in the previous period between a firm in the representative household and a worker from another household. For this match, the representative household instructs the firm to offer a wage rate w in the current period. We assume that the firm makes a take-it-or-leave-it offer, and so the wage offer gives the worker zero surplus.[12] The wage payment to the worker adds to the worker's household's money holdings at the end of the current period. Thus, the utility value of the wage payment to the worker's household is $w\widehat{\omega}_m$, and the worker's surplus is $(w\widehat{\omega}_m - \varphi)$. Setting this surplus to zero, we have

$$w = \varphi / \widehat{\omega}_m \ . \tag{2.6}$$

The worker will accept the wage offer: Although the worker receives zero surplus, a firm can always increase the wage rate slightly to induce the worker to accept the offer with probability one. Similarly, the wage rate in the current period for a worker in the representative household employed by other households is $\widehat{w} = \varphi/\omega_m$. Notice that the tightness of the labor market affects the wage rate only indirectly through its equilibrium effect on the shadow value of money, because the matching rates appear only in the laws of motion of the endogenous state variables.

Now consider a match in the goods market between a buyer from the representative household and a seller from another household. The decision by the buyer's household is to prescribe the quantity of money that the buyer gives to the seller, x, and the quantity of goods, q, that the buyer asks the seller to provide. In contrast to the labor market, making the assumption of one side taking all in a match in the goods market would lead to trivial results. The buyer must obtain positive surplus in order for money to have positive value, and the seller must obtain positive surplus in order to have incentive to accumulate inventory.

To give positive surplus to both sides of the match, we use the following trading protocol: The buyer makes a take-it-or-leave-it offer, but his offer is constrained by the requirement that it should give the seller a surplus greater than or equal to $\theta\widehat{\Delta}$, where $\theta \in (0, 1)$ and $\widehat{\Delta}$ is the total surplus in a similar match. This trading protocol is a short-cut to a more elaborate setting where the buyer and seller can both propose the trading quantities prescribed by their households, with the seller being chosen to propose with probability θ and the buyer with probability $(1 - \theta)$ in each round of bargaining (e.g., Shi, 2001). It is useful to emphasize that these trading quantities

[12]We make this assumption in order to simplify the model. If Nash bargaining is assumed, instead, there will be an additional unidenfiable parameter in the calibration, i.e., the worker's bargaining power. This bargaining power affects workers' search behavior significantly only when workers' search effort is elastic. Since workers' search effort is quite inelastic in the data (and is fixed in the model), using Nash bargaining to determine wages may have only small effects on the model's quantitative results.

are prescribed before the match is formed and hence are committed to by the households. In Section 6.2 we will examine the alternative setting where the trading quantities are determined ex post by Nash bargaining.

A successful trade in the goods market gives the seller x units of (normalized) money, whose value to the seller's household is $\widehat{\omega}_m x$. The seller's opportunity cost of the trade is the value of the goods traded. If the seller keeps the goods, a fraction δ_k will depreciate and the remaining will have a value $(1-\delta_k)\widehat{\omega}_k q$. Thus, the seller's expected surplus from the trade is $[\widehat{\omega}_m x - (1-\delta_k)\widehat{\omega}_k q]$. (The surplus is evaluated with marginal values of the traded objects because each member is infinitesimal in the household.) Since the buyer makes the offers, he will push the seller's surplus down to the minimum, $\theta\widehat{\Delta}$. Thus, the quantities of trade satisfy

$$q = \frac{\widehat{\omega}_m x - \theta\widehat{\Delta}}{(1-\delta_k)\widehat{\omega}_k}. \tag{2.7}$$

The buyer's surplus of trade is $[U'(c)q - \omega_m x]$, and the total surplus of trade in the match is $\Delta = [U'(c) - (1-\delta_k)\widehat{\omega}_k]q$.

Analogous to Δ, the total surplus in a trade match between an arbitrary buyer and an arbitrary seller is $\widehat{\Delta}$. That is,

$$\widehat{\Delta} = \left[U'(\widehat{c}) - (1-\delta_k)\widehat{\omega}_k\right]\widehat{q}. \tag{2.8}$$

We now use dynamic programming to formulate the representative household's decision problem. In each period, the endogenous state variables for the household are (m, k, l) and the choice variables are $(q, x, w, c, e, v, m_{+1}, k_{+1}, l_{+1})$. Taking the aggregate variables as given, the representative household solves:

$$(PH) \quad J(m, n_p k, n_p l) = \max \left\{ \begin{array}{r} U(c) - n_p(1+\widehat{l}\,)\varphi - b\Phi(e) - n_p H(v) \\ +\beta E J(m_{+1}, n_p k_{+1}, n_p l_{+1}) \end{array} \right\}.$$

The constraints are (2.6), (2.7) and the following:

$$m/b \geq x; \tag{2.9}$$

$$\widehat{G}_b beq + d\widehat{c} \geq c; \tag{2.10}$$

$$(1-\delta_w)l + \widehat{\mu}v \geq l_{+1}, \quad \delta_w \in (0,1); \tag{2.11}$$

$$(1-\delta_k)\left[k + f(l) - \widehat{G}_s s(k)\widehat{q} - \frac{d\widehat{c}}{n_p} - \widehat{f}I\right] \geq k_{+1},\ \delta_k \in (0,1); \tag{2.12}$$

$$\frac{1}{\gamma_{+1}}\left\{m + (\gamma_{+1}-1) + \left[\widehat{G}_s n_p s(k)\widehat{x} - \widehat{G}_b bex\right] + n_p(\widehat{w}\widehat{l} - wl)\right\} \geq m_{+1}. \tag{2.13}$$

In addition, there are non-negativity constraints on (x, l, k, m), which we omit.

The constraints (2.6) and (2.7) come from the earlier discussion on the terms of trade. Constraint (2.9) is the money constraint in a trade match. It must be satisfied for every buyer in a trade match because household members are temporarily separated in the exchange.

Constraint (2.10) states that the household's consumption consists of goods obtained from monetary and non-monetary trades in the period. The amount from all monetary trades is the amount in each trade match, q, times the number of trade matches. The amount from non-monetary trades is $d\widehat{c}$, where $d \in (0,1)$ is a constant. This amount is introduced purely for the convenience of calibration and it is taken as given by the individual household.

Constraint (2.11) is the law of motion of employment in each firm in the household. It states that the number of workers employed in the firm next period will consist of retained workers and new hires in the current period. Similarly, constraint (2.12) is the law of motion of inventory held by each firm in the household. It states that inventory per seller at the beginning of next period will consist of inventory that is not depreciated at the end of the current period. During the current period, new output adds to inventory and sales reduce inventory. The amount of goods sold for money by each seller is $\widehat{G}_s s(k)\widehat{q}$ and the amount of non-monetary sales is $d\widehat{c}/n_p$. Again, for the convenience of calibration, we introduce an amount of fixed investment, $\widehat{f}I$, where $I \in (0,1)$ is a constant. The individual household takes this amount as given.

Finally, (2.13) is the law of motion of the household's money holdings. It states that changes in the household's money holdings between two adjacent periods come from the monetary transfer at the beginning of next period, $(\gamma_{+1}-1)$, buying and selling in the current goods market, and buying and selling of labor services. The factor $1/\gamma_{+1}$ appears on the left-hand side of the constraint because money holdings are normalized by the aggregate money stock.

2.4. Optimal Decisions

To find the conditions for optimal choices, note first that (2.11)–(2.13) all hold with equality, provided that $(\omega_m, \omega_k, \omega_l)$ are positive. Use these equalities to substitute for m_{+1}, k_{+1} and l_{+1} in the objective function of (PH). Also, (2.10) holds with equality and we use it to substitute for c. Next, substitute q from (2.7). Let the shadow price of (2.9) be $\widehat{G}_b be\lambda$, where $\widehat{G}_b be$ is the number of matches in which the household's buyers face the constraint (2.9). The first-order conditions of v, e and x are as follows:

$$\text{for } v\text{:} \quad H'(v) = \widehat{\mu}\omega_l; \tag{2.14}$$

$$\text{for } e\text{:} \quad \Phi'(e) = \widehat{G}_b[U'(c)q - \omega_m x]; \tag{2.15}$$

$$\text{for } x\text{:} \quad U'(c)\frac{\widehat{\omega}_m}{(1-\delta_k)\widehat{\omega}_k} = \omega_m + \lambda. \tag{2.16}$$

Condition (2.14) requires that the marginal cost and the expect marginal benefit of a vacancy be equal to each other. The amount $\widehat{\mu}\omega_l$ is the expected marginal benefit of a vacancy, since an additional vacancy will result in a match with probability $\widehat{\mu}$ which will increase the firm's employment next period. Similarly, (2.15) requires that the marginal cost to a buyer from increasing search intensity be equal to the marginal benefit, the latter of which is the buyer's surplus from a trade times the probability of a match.

Condition (2.16) is the optimal condition for the quantity of money traded in a match, x. To interpret it, note from (2.7) that the additional quantity of goods that a buyer can entice the seller to produce for one additional unit of money is $\widehat{\omega}_m/[(1-\delta_k)\widehat{\omega}_k]$. Thus, the left-hand side of (2.16) is the buyer's value of a marginal unit of money spent in a trade. The right-hand side is the cost of money, which consists of the opportunity cost of giving up an additional unit of money, ω_m, and the resulting cost of facing a tighter trading constraint (2.9), λ.

We can also derive the envelope conditions for money holdings (m), employment per firm (l), and inventory (k). Moving the time index forward by one period, these conditions are:

$$\omega_m = E\left[\frac{\beta}{\gamma_{+1}}\left(\omega_{m_{+1}} + \widehat{G}_{b_{+1}} e_{+1}\lambda_{+1}\right)\right]; \tag{2.17}$$

$$\omega_l = \beta E \left\{(1-\delta_w)\omega_{l_{+1}} + \left[(1-\delta_k)\omega_{k_{+1}} f'(l_{+1}) - \omega_{m_{+1}} w_{+1}\right]\right\}; \tag{2.18}$$

$$\omega_k = \beta E \left\{(1-\delta_k)\omega_{k_{+1}} + \widehat{G}_{s_{+1}} s'(k_{+1})[\omega_{m_{+1}} \widehat{x}_{+1} - (1-\delta_k)\omega_{k_{+1}} \widehat{q}_{+1}]\right\}. \tag{2.19}$$

Again, the expectations are conditional on the information available in the current period after the shocks are realized.

Since (2.17) – (2.19) have similar interpretations, we explain only (2.19). This condition characterizes optimal inventory in the next period, k_{+1}. If a seller has one additional unit of inventory at the beginning of next period, the seller's matching probability will increase by $\widehat{G}_{s_{+1}} s'(k_{+1})$. Once in a match, the seller's surplus will be $[\omega_{m_{+1}} \widehat{x}_{+1} - (1-\delta_k)\omega_{k_{+1}} \widehat{q}_{+1}]$. Thus, the service generated by one unit of inventory next period is equal to the product of these two terms. In addition, each unit of inventory will have a value $(1-\delta_k)\omega_{k_{+1}}$ two periods later. The right-hand side of (2.19) is the expected sum of this future value and the service, properly discounted. If k_{+1} is chosen optimally, then this sum is equal to the marginal value of capital, ω_k, as (2.19) requires.

It is clear that inventory generates a positive service only if $s' > 0$ and if a seller obtains a positive share (θ) of the match surplus. If either $s' = 0$ or $\theta = 0$, then inventory is positive only when the expected value of inventory grows at a gross rate $1/(1-\delta_k)$. In this case, there cannot be a stationary level of inventory in equilibrium.

3. Equilibrium and Velocity

The following defines a symmetric search equilibrium:

Definition 3.1. *For any given initial state (m_0, k_0, l_0) and the exogenous shock processes, a symmetric monetary search equilibrium consists of each household's choice variables j and other households' choices $\widehat{j}$, where $j \equiv (c, x, q, w, e, v, m_{+1}, l_{+1}, k_{+1})$, such that (i) j solves (PH) under given $\widehat{j}$ and (m, l, k); (ii) $j = \widehat{j}$; and (iii) the values of $\omega_m m_{+1}$, $\omega_l l_{+1}$, and $\omega_k k_{+1}$ all lie in $(0, \infty)$.*

The requirements (i) and (ii) are self-explanatory. Notice that symmetry implies $m = 1$ in equilibrium. With symmetry, we will suppress the hat on aggregate variables. The condition (iii) requires that money, employment and inventory all have positive values in equilibrium in order for the analysis on these stock variables to be meaningful. The condition also requires that the total value of each stock variable be bounded in order for the first-order conditions to characterize the optimal choices.

We assume that the trading restriction (2.9) binds in all periods, i.e., $\lambda > 0$. The reason for making this assumption is that, if $\lambda = 0$, then money only performs the role of a store of value in that period but not the role of a medium of exchange. In the quantitative exercises we will ensure $\lambda > 0$, for which the restriction $\gamma > \beta$ is imposed.[13]

With the restriction $\lambda > 0$ and the result $m = 1$, we have $x = 1/b$ in equilibrium. Thus, the price level, normalized by the aggregate money stock, is $p = x/q = 1/(bq)$. The gross rate of inflation between two adjacent periods is $p_{+1}M_{+1}/(pM) = \gamma_{+1}q/q_{+1}$.

We can reduce the equilibrium system to a dynamic system of five variables $(l, k, v, \omega_k, \omega_m)$. This is done in the following steps. First, we substitute $x = 1/b$ and express the matching rates as $\mu = \mu(v)$, $G_b = G_b(e, k)$, and $G_s = G_s(e, k)$. Second, we substitute $w = \varphi/\omega_m$ from (2.6) and $\omega_l = h(v)$ from (2.14), where

$$h(v) \equiv H'(v)/\mu(v). \tag{3.1}$$

Third, we use (2.10), (2.7), (2.16) and (2.15) to obtain:

$$c = \frac{1}{1-d} G_b(e, k) beq, \tag{3.2}$$

$$\omega_m = bq \left[\theta U'(c) + (1-\theta)(1-\delta_k)\omega_k\right], \tag{3.3}$$

$$\frac{\lambda}{\omega_m} = \frac{U'(c)}{(1-\delta_k)\omega_k} - 1, \tag{3.4}$$

$$\Phi'(e)/G_b(e, k) = (1-\theta)q[U' - (1-\delta_k)\omega_k]. \tag{3.5}$$

[13]The assumption $\lambda > 0$ in all periods strengthens our results since it underestimates the variability of velocity in the model. If $\lambda = 0$ in some periods, velocity would be more volatile between periods. However, the extent of this underestimation is small because the data has $\gamma > \beta$ in most periods.

Jointly solving (3.2) – (3.5), we express (c, e, q, λ) as functions of (k, ω_m, ω_k). Finally, substituting the above results into (2.11), (2.12) and (2.17) – (2.19), we have the following dynamic system:

$$\left.\begin{aligned}
&l_{+1} = (1-\delta_w)l + v\mu(v), \\
&k_{+1} = (1-\delta_k)\left[k + (1-I)f(l) - \tfrac{1}{1-d}G_s(e,k)sq\right], \\
&\omega_k = \beta(1-\delta_k)E\left\{\omega_{k_{+1}}\left[1 + \theta q_{+1}G_s(e_{+1},k_{+1})s'(k_{+1})\tfrac{\lambda_{+1}}{\omega_{m_{+1}}}\right]\right\}, \\
&h(v) = \beta E[(1-\delta_w)h(v_{+1}) + (1-\delta_k)\omega_{k_{+1}}f'(l_{+1}) - \varphi], \\
&\omega_m = E\left\{\tfrac{\beta}{\gamma_{+1}}[\omega_{m_{+1}} + G_b(e_{+1},k_{+1})e_{+1}\lambda_{+1}]\right\}.
\end{aligned}\right\} \tag{3.6}$$

In this system, (k, l) are predetermined variables and others are jump variables.

Consumption velocity of money is defined as $V_c = pc/m$. Because $p = 1/(bq)$ and $m = 1$, then (3.2) implies

$$V_c = \frac{eG_b}{1-d} = \frac{1}{1-d}g_0e^{\xi}\left[\frac{n_p}{b}s(k)\right]^{1-\xi}. \tag{3.7}$$

Velocity is proportional to a buyer's trading frequency and so it depends only on the extensive margin of trade. In turn, the extensive margin depends only on buyers' search intensity, e, and sellers' inventory, k. Given e and k, velocity does not depend on the intensive margin of trade, i.e., not on the quantity of goods traded in each match. The reason for this result is simple. Because prices are fully flexible in this model, any change in q will result in opposite changes in consumption and the price level, which will leave the product pq and hence velocity independent of q for given (e, k). Thus, changes in q can affect velocity only indirectly by affecting e and k.

It is useful to anticipate the immediate impact of shocks on velocity. Consider first a positive shock to the money growth rate. Because inventory is predetermined, the immediate impact of this shock on velocity goes through buyers' search intensity. A buyer will search more intensively if and only if the shock increases the buyer's surplus in a trade. This surplus is the product of the quantity of goods traded in a match, q, and the buyer's surplus per unit of good, $(1-\theta)[U' - (1-\delta_k)\omega_k]$ (see (3.5)). Higher money growth is likely to affect these two dimensions of a buyer's surplus in opposite directions and hence to have an ambiguous impact on velocity. On the one hand, higher money growth increases anticipated inflation and reduces the real purchasing power of money. This effect will reduce the quantity of goods that a buyer obtains in a trade, q. On the other hand, higher money growth will increase the buyer's surplus per unit of good. This is because consumption is more valuable when consumption is low. As higher money growth reduces the household's consumption, an additional unit of good obtained from the trade has a higher benefit to the household. This indirect effect is stronger if the household is more risk averse. Thus, the higher the relative risk aversion, the stronger the indirect (positive) effect of money growth on a buyer's surplus, and the more likely that a buyer will search intensively.

The impact of a positive technology shock on velocity is opposite to that of a money growth shock. By increasing the supply of goods, a positive technology shock increases q and reduces the marginal value of goods to the consumer. Thus, when the shock is realized, search intensity and velocity will fall if the relative risk aversion is high and will rise otherwise.

After the immediate impact, the shock will continue to affect search intensity, velocity and output in future periods. This propagation mechanism will depend on a number of factors such as inventory and labor market search. We will explore this mechanism later in Section 5.3.

4. Calibration and Computation

We calibrate the model to the quarterly US data (see Appendix A for a description of the data). The sample covers the period 1959:II – 1998:III and is filtered using the HP filter. We also include the results for the sub-sample 1959:II – 1988:I, which Hodrick et al. (1991) examined.

For the shocks, we assume that γ and $\ln A$ obey a vector auto-regressive (VAR) process:

$$\begin{pmatrix} \gamma_{t+1} \\ \ln A_{t+1} \end{pmatrix} = N_1 + N_2 \begin{pmatrix} \gamma_t \\ \ln A_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{m,t+1} \\ \varepsilon_{A,t+1} \end{pmatrix}, \tag{4.1}$$

where N_1 is a 2×1 vector and N_2 a 2×2 matrix. We call ε_m the shock to money growth and ε_A the productivity shock. As clarified in the introduction, we impose no restriction on the VAR estimation, and so ε_m is not necessarily a shock to monetary policy. Also as explained in the introduction, we use $M2$ as the monetary aggregate to compute γ. To construct the time series of log productivity, we interpret $f(l) = Al^{\alpha}$ as aggregate labor income and calculate it to be 64%

of GDP (see Christiano, 1988). Normalize the steady state level of A as $A^* = 1$ and identify α through the procedure described later. Then, using the data of (f, l), we obtain the time series of log productivity as $\ln(A_t) = \ln(f_t) - \alpha \ln(l_t)$.

The results of the VAR estimation are reported in Table 1. As the table shows, the first-order specification is good. Also, $\ln A$ is positively correlated with lagged money growth.

Table 1 here.

The above procedure differs from that in Hodrick et al. (1991, Table 2) in two aspects. First, Hodrick et al. examine endowment economies, rather than productive economies, and so output fluctuations are caused entirely by exogenous endowment shocks. In our model, output can be caused by endogenous fluctuations in employment as well as exogenous productivity shocks. Second, we use log productivity in the VAR specification, but Hodrick et al. use endowment growth in the specification. This difference reflects the difference in the treatment of the data. Hodrick et al. do not filter the data, in which case it is appropriate to assume that the growth rate of endowment is stationary. In contrast, we filter the data, and so it is appropriate to assume that the level (or the log level) of productivity is stationary.[14] Because of these differences, the variability of velocity is comparable between the models only as a percentage of the variability of velocity in the corresponding data to which the model is calibrated.

To calibrate the model, we choose the following functional forms:

$$U(c) = \frac{c^{1-\eta} - 1}{1-\eta}; \quad s(k) = \frac{k^{1-\varepsilon_k} - 1}{1-\varepsilon_k};$$

$$\Phi(e) = \varphi(\varphi_0 e)^{1+\frac{1}{\varepsilon_e}}; \quad H(v) = H_0 v^2.$$

Here, η, ε_k, ε_e, φ, φ_0 and H_0 are all positive constants. The constant ε_k measures the curvature of a seller's searching intensity function. In the search cost function, φ is the disutility of employment, φ_0 is the efficiency units of a buyer's search intensity relative to a worker's time, and ε_e measures the elasticity of buyers' search intensity.

[14] In a previous version of this paper, we calibrated the model to the unfiltered data and still found that the variability of velocity is much larger than in Hodrick et al. (1991).

We identify the model separately for the sample 1959:II – 1998:III and the sample 1959:II – 1988:I. The identified parameter values from the shorter sample are put in parentheses in the following description. Set $\beta = 0.9952$ (0.9958) to match the sample mean of the quarterly real interest rate, 0.4809% (0.4191%). The gross rate of money growth in the steady state matches the sample average $\gamma^* = 1.01724$ (1.02014). Set $I = 0.269$ to match the ratio of fixed investment to output in the data (see Christiano, 1988). Set $d = 0.25$, which is a realistic number for the fraction of purchases made through non-monetary trades.[15] The share of vacancies in the formation of labor market matches is set at $\psi = 0.6$ and the job separation rate at $\delta_w = 0.06$. Both of which are consistent with the estimates by Blanchard and Diamond (1989).

The parameters $(\eta, \xi, \theta, \varepsilon_e, \varepsilon_k, B)$ cannot be identified. To address this problem, we will first set these parameters to certain values in the benchmark case and then examine the sensitivity of the results to changes in these parameters. The relative risk aversion is set at $\eta = 4$ and allowed to vary in the range [0.2, 8]. These values are comparable with those in Hodrick et al. (1991). Also, set the share of buyers' search in the formation of matches to $\xi = 0.8$, the sellers' surplus share in a match to $\theta = 0.2$, the elasticity of buyers' search intensity to $\varepsilon_e = 2$, the parameter in a seller's search intensity function to $\varepsilon_k = 13$, and the ratio of buyers to sellers to $B = 0.5$. The sensitivity analysis on $(\xi, \theta, \varepsilon_e, \varepsilon_k, B)$ will appear in Section 6.1.

The remaining parameters, including the parameter α in the production function, are identified by matching the model's predictions with the following facts. (i) The labor participation rate is 0.6282 (0.6148) and the unemployment rate is 0.0605 (0.0611); (ii) The inventory/output ratio is 0.9 and the inventory investment/output ratio is 0.0065; (iii) Income velocity of money is 1.7440 (1.6882); (iv) The share of labor income in output is 0.64 (Christiano, 1988) and the hiring cost is 2% of the wage cost (which is in the range surveyed by Hamermesh 1993); (v) The shopping time of the population is 11.17% of the working time and the working time is 30% of agents' discretionary time (Juster and Stafford, 1991). In Appendix B we detail the procedure to use the above restrictions to solve the parameters.

[15] Avery et al. (1986) report that US households perform about 82% of their purchases with money.

The parameter values are listed in the upper panel of Table 2. The lower panel of Table 2 reports steady state values of some endogenous variables.

Table 2 here.

To compute the model, we use the standard method described by Blanchard and Kahn (1980). That is, we linearize the equilibrium dynamic system in Section 3 around the steady state and find the corresponding saddle path that is consistent with rational expectations.

We simulate the model and compute the unconditional moments of some key variables and their correlations that Hodrick et al. (1991) examine.[16] The draws of the shocks are restricted to satisfy $\gamma > \beta$, so as to ensure $\lambda > 0$. As in Hodrick et al., we measure the variability (or volatility) of velocity by the coefficient of variation, defined as follows:

$$cv(V_c) = \frac{\sigma(V_c)}{E(V_c)} \times 100. \tag{4.2}$$

Also, define the following variables:

inflation rate:	$\pi = \gamma q_{-1}/q - 1$
real interest rate:	$r = U'(c_{-1})/[\beta U'(c)] - 1$
nominal interest rate:	$i = \gamma\omega_{m_{-1}}/(\beta\omega_m) - 1$
consumption growth:	$g_c = c/c_{-1}$

These definitions are standard. Despite the fact that our model does not have asset markets, interest rates are still meaningful concepts here. For example, if a government can introduce nominal bonds that are not transferable between agents, then the interest rate on such bonds will be the one defined above. Similarly, if each household has a technology to store consumption goods, then the real interest rate defined above will serve as a lower bound on the rate of return to such storage.

5. Model Predictions

Tables 3.1 and 3.2 report the unconditional moments of variables, where the standard deviations of the moments over the simulations appear in brackets. In these tables, we choose five different

[16] The total number of simulation is 1000. Standard deviations of second order moments are calculated over these rounds of simulation.

values of the relative risk aversion, η (0.2, 2, 4, 6, and 8), where $\eta = 4$ is the benchmark value.[17] Means and standard deviations of inflation and interest rates are almost constant, and so they are not reported here. It is clear from the two tables that the results are very similar for the period 1959:II - 1988:I (which was examined by Hodrick et al., 1991) and for the period 1959:II - 1998:III. In the following discussion, we refer only to the results for the longer period.

Tables 3.1 and 3.2 here.

5.1. The Mean and Variability of Velocity

The mean of consumption velocity matches the data well. For a large range of the relative risk aversion (from 0.2 to 8), the mean of velocity varies only slightly from 1.2238 to 1.2240, which is close to the observed mean, 1.2702. The reason for the close match is that we calibrated the steady-state income velocity to that in the data. This procedure yields a realistic mean of consumption velocity, provided that the ratio of consumption to output is realistic and stable.

Our model also explains a large fraction of the variability of velocity in the data. In the benchmark case ($\eta = 4$), for example, the coefficient of variation in velocity is about 90% of the value in the data. Even for η as low as 0.2, the model generates 57% of the variability of velocity in the data. As the relative risk aversion increases, velocity becomes more volatile. When η is equal to or greater than 6, the model even generates higher variability of velocity than in the data. In contrast, Hodrick et al. (1991, Table 6) were able to explain at most 40% of the variability in the data. This maximum variability was obtained under extreme parameter values $\eta = 9.5$ and $\beta = 0.975$, which imply a quarterly real interest rate (2.6%) that is much higher than the sample value (0.42%). We choose β to match the observed mean of the real interest rate. Even with this realistic value of β and a smaller η, our model is able to capture a much higher percentage of the variability in the data.

Of course, a high variability of velocity is meaningful only when it is viewed relatively to the

[17] When investigating the sensitivity of the numerical results to one parameter, we repeat the procedure in Appendix B to identify other parameters again. So, a change in one parameter may entail changes in other parameters in order to satisfy the restrictions in the identifying procedure. If this leads to a change in the value of α, we reconstruct the series $\{lnA_t\}$ for the VAR estimation.

volatility of other aggregate variables such as output. Volatile velocity in our model is not caused by artificially setting the entire model to be more volatile than in the data. To see this, we include the coefficient of variation of output, $cv(y)$, in Tables 3.1 and 3.2. In all simulations, output in the model is less volatile than in the data. In the benchmark case ($\eta = 4$), the model captures about 60% of the volatility of output in the data.

Velocity is volatile in our model because it is determined by the extensive margin of trade. As households change search intensity to keep consumption smooth, the extensive margin of trade can respond to shocks significantly. As explained near the end of Section 3, a household changes search intensity more significantly when it is more risk averse. Thus, it is not surprising to see the result in Table 3.2 that velocity is more volatile when the risk aversion is higher.

5.2. Correlations between Variables

To ensure that the high variability of velocity does not come at the expense of the model's performance in other dimensions, we examine the correlations between the key variables. These correlations have the correct signs and their magnitudes are comparable with those in the data, provided that the relative risk aversion is not very low. First, velocity is negatively correlated with consumption growth and positively with the nominal interest rate. Second, inflation is positively correlated with the nominal interest rate and negatively with the real interest rate. In contrast, some of these correlations are unrealistic in Hodrick et al. (1991, Table 6). In particular, the correlation between velocity and consumption growth in their paper ranges from -0.069 to 0.664, which is very different from the value in the data (-0.3537).

The negative correlation between velocity and consumption growth is important because it is indicative of how the extensive and intensive margins of trade respond to the shocks. Recall that the extensive margin is the trading frequency and the intensive margin is the quantity of goods traded in each match, q. While velocity depends only on the extensive margin, consumption depends on both margins. The negative correlation between velocity and consumption growth reflects the fact that these two margins of trade often respond to a shock in opposite directions. For example, when a positive shock to productivity increases consumption, the households reduce buyers' search intensity because there is not much need to search intensively when goods are abundant. Similarly, when a positive shock to money growth reduces consumption by reducing the purchasing power of money, the households try to smooth consumption by increasing buyers' search intensity. When the relative risk aversion is high, this motive of using search intensity to smooth consumption is strong and the resulting effect on the extensive margin dominates the effect on the intensive margin.

In traditional models, such as the ones in Hodrick et al. (1991), the extensive margin of trade is not important for the equilibrium. It is then not surprising that these models have difficulty to generate a significantly negative correlation between velocity and consumption growth.

5.3. Propagation of the Shocks

Search intensity and the supply of goods play important roles in the propagation of shocks. To assess this propagation mechanism, we study the dynamic effects of the shocks. We do this by first analyzing the cross-correlations between some key variables and then presenting the impulse responses of the equilibrium to the shocks.

In Table 4 we present the cross-correlations of search intensity and inventory with log-productivity and money growth. The following features of the cross-correlations are noteworthy. First, search intensity is negatively correlated with past, present and future productivity, while positively correlated with past, present and future money growth. Inventory is positively correlated with past, present and future productivity, while negatively correlated with past, present and future money growth. Second, the highest correlation between money growth and search intensity is the contemporaneous correlation. This indicates that search intensity responds to a money growth shock by the most at the time of the shock. In contrast, the highest correlation between search intensity and productivity is between current intensity and one-period lagged productivity, which indicates a hump-shaped response of search intensity to a negative productivity shock. Third, consistent with the data, the highest correlations between inventory and the shocks are between current inventory and the shocks with some lags. This delay in the peak response of inventory occurs because it is costly to build up inventory.

Table 4 here.

Figure 3 depicts the impulse responses of the equilibrium to a positive shock to productivity, while the money growth shock is maintained at zero. These impulse responses reveal the following propagation mechanism. At the impact of the shock, the supply of goods increases, which increases consumption and reduces the need to search. As a result, velocity falls. Some of the increased supply of goods in the current period ends up in next period's inventory. This increased inventory and the persistence in productivity will keep the supply of goods in the next period above the steady state. As a result, consumption in the next period will be above the steady state. Households will continue to reduce search intensity and velocity will continue to fall. This pattern continues for several periods until productivity falls back toward the steady state sufficiently. At that point of time, the supply of goods and consumption have dissipated sufficiently toward the steady state. Search intensity starts to rise, which induces velocity to rise toward the steady state. In this adjustment, search intensity, inventory and velocity all have hump-shaped responses, while output and consumption adjust monotonically.

An unconventional feature of this adjustment is that employment responds *negatively* to a positive productivity shock. Although there is an on-going debate on whether this negative response is indeed present in the US data (e.g., Gali, 1999, and Christiano et al., 2003), its presence in our model seems intuitive. A positive productivity shock makes goods more abundant and reduces the expected match surplus per trade for a seller. As a result of this lower surplus, firms reduce employment. Employment continues to fall in the first few periods of the transition as buyers' lower search intensity further reduces a seller's expected match surplus. Only after several periods does employment start rising toward the steady state.

Figure 3 and Figure 4 here.

Figure 4 depicts the impulse responses of the equilibrium to a positive shock to money growth, while the productivity shock is maintained at zero. Supporting the discussion in Section 3, the shock immediately reduces consumption, increases the incentive to search, and increases velocity immediately. These immediate responses are qualitatively opposite to those under a positive productivity shock. However, one period after the shock and onward, the responses of the equilibrium are similar to those under a positive productivity shock. In particular, in the second period after the shock, consumption will jump above the steady state, while search intensity and velocity will fall below the steady state. Thereafter, the adjustment follows the same pattern as the one depicted in Figure 3.

The reversal of the adjustment in period 2 is caused by two features of the estimated VAR structure. First, the money growth shock is not persistent. One period after the shock, money growth returns to levels that are very close to the steady state. Second, the estimated VAR has a positive correlation between current productivity and past money growth. Thus, a positive shock to current money growth increases future productivity. These induced changes in productivity are the driving forces of the responses one period after the shock.

Taken together, Figure 3 and Figure 4 show that changes in productivity, either directly or indirectly through their effect on money growth, exert a dominant force on velocity and its co-movement with consumption and output.

5.4. The Roles of the Two Shocks and Labor Market Search

In this subsection, we investigate the roles of costly search and the shocks. First, productivity shocks are necessary for generating sufficient volatility in velocity. To illustrate this point, we fix productivity at the steady state level and simulate the model. The results are reported in the top panel in Table 5. As in the model with both shocks, money growth shocks alone still generate negative correlations between velocity and consumption growth, and between inflation and the real interest rate. However, velocity is much less volatile than with productivity shocks. For example, with $\eta = 4$, the coefficient of variation in velocity is about 50% of that in the data, as opposed to 90% when there are productivity shocks. Also, the model without productivity shocks can only explain a small fraction (about 10%) of the volatility of output in the data.

Second, money shocks are necessary for explaining some basic features of the data. In the middle panel of Table 5, we report the simulation results with only productivity shocks, where the money growth rate is fixed at the steady state level. Although velocity and output are still volatile in the absence of money growth shocks, the correlation between inflation and the nominal interest rate is negative, which is counter-factual. Also, inflation and the real interest rate are almost perfectly and negatively correlated with each other. The corresponding correlation in the data is much smaller.

Table 5 here.

Third, the non-Walrasian feature of the goods market, modeled as search, is important for generating the variability of velocity in the model. This is simply because velocity is determined by the extensive margin of trade. If the goods market were Walrasian, then the extensive margin of trade would not be important for velocity and hence velocity would not be volatile.

Finally, labor market search is important for the model's performance. To see this, we compute the model under the alternative assumption that the labor market clears in every period in the Walrasian style. That is, the following equation always holds:

$$(1-\delta_k)\omega_k f'(l) = \omega_m w = \varphi. \tag{5.1}$$

This equation determines the level of employment in each firm, l. The first part of this equation is the marginal product of labor and the second part is the wage rate, both being valued in utility. In this alternative economy, the dynamics of (k, ω_k, ω_m) still obey (3.6) in Section 3, but the equation for l is replaced by (5.1) and the equation for v is no longer relevant. Calibrating this model to the data (1959:II – 1998:III), we compute the moments.[18]

The results are reported in the bottom panel in Table 5. Without labor market search, both velocity and output are much less volatile than with labor market search. Consider the case $\eta = 4$, for example. Without labor market search, the model captures about 70% of the volatility of velocity in the data, as opposed to 90% when there is labor market search, and 40% of the volatility of output in the data, as opposed to 60% when there is labor market search.

[18] In particular, the identifying restriction that wage income is 64% of output yields $\alpha = 0.64\,(1 + \theta\lambda^*/\omega_m^*)$.

To explain why velocity is less volatile when labor market search is shut down, consider a positive shock to money growth. Suppose that buyers increase search intensity, as in the case with labor market search. Because high search intensity generates a positive externality to sellers, a firm's expected surplus from the goods market increases. Anticipating this higher surplus, firms will increase employment. Thus, output and the supply of goods will likely rise immediately when the shock is realized. This increase in the supply of goods will reduce buyers' incentive to search. That is, search intensity will respond by less to the shock, and hence velocity will be less volatile, than if search is required in the labor market.

6. Sensitivity Analysis

In this section we examine the sensitivity of the quantitative results to some parameters and to the bargaining rule in the goods market.

6.1. Sensitivity to the Parameters

So far we have chosen the following parameters exogenously: the buyer/seller ratio in the goods market, B, the seller's surplus share in a match, θ, the elasticity of buyers' search intensity, ε_e, the share of buyers' search intensity in the matching function in the goods market, ξ, and the parameter in sellers' intensity function, ε_k. Now we examine the sensitivity of the results to these parameters. To do so, we change each of these parameters separately and calibrate the model to the data again.[19] The relative risk aversion is kept at the benchmark value, $\eta = 4$. Since the results are similar when the model is calibrated to the short sample and to the long sample, we report only the results for the long sample in Table 6.

Table 6 here.

The variability of velocity is insensitive to changes in the buyer/seller ratio, B. To explain this insensitivity, note that an increase in the number of buyers creates two types of externality. One

[19] Ideally, we would like to pin down the value of θ by matching the mark-up ratio in our model to that in the data. Unfortunately, the markup in our model varies too little with respect to θ: When θ varies from 0.01 to 0.99, the markup almost remains constant at 0.44. This value of the markup lies in the range, $(0.4, 0.7)$, which Domowitz et al. (1988) estimated from the US data but it is much smaller than the value (1.5) which Hornstein (1993) estimated.

is to increase the congestion for buyers. This negative externality reduces the number of matches per buyer and reduces velocity. The other externality created by an increase in the number of buyers is to increase each seller's matching rate, which induces sellers to increase production and inventory. This positive externality increases the number of matches per buyer and increases velocity. Similar to Hosios (1990), the two externalities cancel out with each other when the two sides of the market are rewarded according to their contributions to the match formation. The latter condition is $\theta = 1 - \xi$, which is satisfied in the benchmark model. As a result, an increase in the number of buyers does not change much the number of matches per buyer or velocity.

The variability of velocity responds positively to an increase in each of the three parameters, θ, ε_e, and ξ, but this response is not very large. This insensitivity is surprising because one would expect that a change in any of these parameters would affect agents' search decisions, and hence velocity, significantly. An important reason for the insensitivity is that a change in these parameters is accompanied by changes in other parameters that are necessary for satisfying the identifying restrictions in Section 4. These accompanying changes offset a large part of the effects on velocity caused by the change in the parameter in the discussion. For example, when buyers' search intensity, ε_e, increases, it becomes responsive to a shock to productivity or money growth. However, to satisfy the identifying restriction on the ratio of shopping time to working time, the parameter in the matching function, g_0, must fall and the tightness of the goods market for buyers must rise. These changes restrain the effect of an increase in ε_e on velocity.[20]

Now consider the sensitivity of the results with respect to ε_k, a parameter in the function $s(k)$ that translates a firm's inventory into its search intensity. Since the function s did not appear in previous search models, we experiment a wide range of values of ε_k, from 0.5 to 16. As ε_k increases from 0.5 to 16, most of the simulated moments, such as the negative correlation between velocity and consumption growth, remain relatively stable. However, the variability of velocity increases significantly with ε_k. This is puzzling because when ε_k increases, the function $s(k)$ is

[20]Another reason for the insensitivity is that some of these parameters, such as θ and ξ, are shares to agents on one side of the goods market. A change in such shares affects the behavior of the two sides of the market in opposite directions. Because velocity depends on both buyers' search intensity and sellers' inventory, the opposite effects largely offset each other.

less elastic with respect to inventory, which should make the number of trades and velocity less (rather than more) volatile. Again, the explanation lies in the calibration process. Each time when ε_k changes, the model is calibrated again to satisfy the identifying restrictions in Section 4 and, in particular, to satisfy Restriction (iii) on income velocity of money. An increase in ε_k requires a large decrease in the steady state value of the function $s(k)$, i.e., s^*. In this case, any given amount of response in s represents a large change relative to the steady state level, and hence a large variation in velocity. This effect dominates the direct effect of ε_k on velocity, and hence generates the positive response of the variability of velocity to ε_k.

Notice that the variance ratio of output to sales decreases when ε_k increases. However, this ratio is much higher than in the data. Even when ε_k is given a seemingly large value, 16, the variance ratio is 3.56. In contrast, Blinder and Maccini (1991) found that the variance ratio does not exceed 1.3 in the US data. In this sense, the benchmark model provided a conservative estimate for the variability of velocity in the model. By increasing ε_k to reduce the variance ratio of output to sales toward a realistic level, the model can generate higher variability of velocity.

6.2. Nash Bargaining

In the analysis so far, we assume that a buyer in a match (in the goods market) makes a take-it-or-leave-it offer but the offer is constrained to give the seller a surplus no less than $\theta\widehat{\Delta}$. Since $\widehat{\Delta}$ is exogenous to the agents in a match, this bargaining protocol has an element of commitment. In this section, we explore the Nash bargaining scheme which eliminates this element. Following the convention, we assume that the threat point of each trader in bargaining is the future value of the assets/goods that the trader has brought into the trade. Thus, if the quantities (q, x) are traded in a match, the buyer's surplus is $[U'(c) - \omega_m x]$ and the seller's is $[\widehat{\omega}_m x - (1 - \delta_k)\widehat{\omega}_k q]$. Let θ now denote the seller's weight in Nash bargaining. The quantities (q, x) are now the solution to the following problem:

$$\max_{(q,x)} \; [\widehat{\omega}_m x - (1 - \delta_k)\widehat{\omega}_k q]^{\theta} \, [U'(c) - \omega_m x]^{1-\theta}, \quad \text{s.t. } x \leq m/b.$$

The main analytical difference between this formulation and the earlier one is that the quantities (q, x) are determined after the households' other decisions, rather than at the same time as other decisions. As a result, these quantities (q, x) are functions of the household's other decisions. To see this, let λ now denote the Lagrangian multiplier of the constraint in the above problem. In a symmetric equilibrium, $\lambda > 0$ if $U' > (1 - \delta_k)\omega_k$, as in the previous formulation. Assume $\lambda > 0$. Then, $x = m/b$. Also, the first-order conditions yield:

$$q = \frac{m}{b}\left[\theta\frac{\omega_m}{U'(c)} + (1-\theta)\frac{\widehat{\omega}_m}{(1-\delta_k)\widehat{\omega}_k}\right]. \tag{6.1}$$

Denote this function as $q = q(m, c)$. Substituting $q(m, c)$ into (2.10), we can solve for $c = C(m, e)$. Then, $q = Q(m, e) \equiv q(m, C(m, e))$.

In this environment, a household choosing consumption must take into account the effect of consumption on the quantity of trade. Since consumption depends on buyers' search intensity, then a household must also consider how the choice of search intensity affects the trading quantities. These effects are summarized by the functions $q(m, c)$ and $Q(m, e)$. To incorporate these changes, we modify the optimization problem (PH) in Section 2.3 by replacing q with $Q(m, e)$, c with $C(m, e)$ and x with m/b. The constraints (2.11) through (2.13) still apply.

Optimal search intensity is given by the following condition:

$$\Phi'(e) = \widehat{G}_b\left[\frac{U'(c)q}{1-\widehat{G}_b beq_c} - \frac{m}{b}\omega_m\right], \tag{6.2}$$

where q_c is the derivative of the function $q(m, c)$ with respect to c. The only difference between this condition and its counterpart in the previous formulation, (2.15), is the presence of the effect q_c. Similarly, the dynamic equations for ω_k and ω_m in (3.6) need be modified. In particular, ω_m now obeys the following dynamic equation:

$$\omega_m = E\left\{\frac{\beta}{\gamma_{+1}}\left[\frac{G_b beq}{1-G_b beq_c}U' + (1-G_b e)\omega_m\right]\right\}. \tag{6.3}$$

We calibrate this alternative model using the same identifying restrictions as in Section 4. For a monetary steady state to exist in this alternative formulation, the relative risk aversion must be small. To see this problem, suppose that a household expects that consumption will increase. The marginal utility of consumption will decrease, which will strengthen the buyers' bargaining position in a match (i.e., $q_c > 0$). In turn, if the household's buyers all bring in more goods, then consumption will indeed rise. This reinforcing structure will reduce the marginal utility of consumption sufficiently when the relative risk aversion is high. However, for money to serve as a medium of exchange, the marginal utility of consumption cannot be too low; otherwise the households would hoard money. Thus, for an interesting steady state to exist, the relative risk aversion must be low in order to limit the strength of the reinforcing structure between consumption and q. When other parameters are set to realistic values, the relative risk aversion must be much lower than one.[21] For a higher value $\eta = 4$ to be consistent with a monetary steady state, the fraction of consumption obtained through non-monetary trades must exceed 0.85. In Table 7 we report the results for two combinations of (η, d).

Table 7 here.

With $\eta = 0.1$ and $d = 0.25$, this model produces lower volatility of velocity than in the benchmark model, despite that it generates much higher volatility of output. In contrast to the benchmark model and to the data, velocity is now positively correlated with consumption growth and negatively with nominal interest rate. Also, the positive correlation between inflation and the nominal interest rate is almost unity, which is much larger than in the data. These results are not surprising. Even in the previous formulation, these counterfactual results can occur with very low relative risk aversion (see the case $\eta = 0.2$ in Table 3.2).

The case with $\eta = 4$ and $d = 0.87$ does better to match the data. In this case, velocity is much more volatile and output is less volatile than in the benchmark model. Notice that the correlation between inflation and the nominal interest rate is negative, rather than positive as in the benchmark model and in the data. This is because monetary transactions generate only a small fraction of consumption in this model, in which case inflation is driven primarily by (negative) productivity shocks rather than monetary shocks.

[21] The same problem seems to arise in Lagos and Wright (2002). In all their calibration exercises, the relative risk aversion is lower than 0.5.

7. Conclusion

We construct a dynamic search model to examine the behavior of velocity. The prominent feature of the model is the presence of costly search in the goods market and the labor market. Incorporating shocks to money growth and productivity, we calibrate the model to the US time series data. Our model captures most of the volatility of velocity in the data. The moments of other endogenous variables are realistic and in particular, the correlation between velocity and consumption growth is negative in the model as in the data. We also find that the model generates persistent propagation of the shocks. Overall, productivity shocks exert a dominant force on velocity and its co-movement with other variables.

The high volatility of velocity and persistent propagation of shocks can both be attributed to costly search. Costly search in the goods market is important because in its presence, velocity is equal to the trading frequency per buyer. This extensive margin of trade depends on buyers' search intensity and sellers' inventory. In the event of shocks, search intensity changes significantly as buyers try to smooth consumption. This creates part of the variability of velocity and initial propagation to the shocks. Costly search in the labor market creates the additional variability of velocity and further propagation. It does so by delaying the response of employment to shocks and attenuating the immediate response of output. As a result, inventory is an important component of the supply of goods. Since shocks affect future inventory by changing current consumption and sales, they generate persistent effects on future search intensity, velocity and output.

It should be noted that money is the only asset in the model. Thus, asset substitutions driven by interest rate changes are not the factor that makes velocity volatile here. Instead, all variations in velocity are caused by the variations in the extensive margin of trade. Despite this deliberate restriction, the model's performance is encouraging. It indicates that search model of money should be taken seriously in quantitative analyses, as well as in theoretical analyses.

Besides the behavior of velocity, our model provides some interesting results on the propagation of the shocks. Given the way in which we identified the shocks, these results should be viewed as preliminary ones that stimulate further investigation. Two of these results are particularly noteworthy. One is that money growth shocks can affect velocity and output persistently. This result raises the hope that shocks to monetary policy may also have persistent effects on real activities. However, since not all changes in money growth are caused by monetary policy shocks, one needs to identify monetary policy shocks before examining the policy effects. Another result is that a positive technology shock causes a persistent decline in employment. Given the recent controversy on the response of employment to technology shocks in traditional models of business cycles (see Gali, 1999, and Christiano et al., 2003), it may be useful to investigate further whether a search model can provide a different perspective on the issue. Again, to investigate this issue thoroughly, we need to identify the shocks more carefully.

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A. Data Sources

Data used in our paper mainly came from the following sources (all seasonally adjusted).

1. Citibase (Acronyms in bracket)

   - M1 Aggregate (FM1), monthly. The quarterly data is calculated from the average of three months.
   - M2 Aggregate (FM2), monthly. The quarterly data is calculated from the average of three months.
   - Real M2 Aggregate (FM2DQ), monthly. The quarterly data is calculated from the average of three months.
   - Nominal interest rates (FYGM3), 3-month Treasury bill yield, monthly. The quarterly data is calculated from the average of three months.
   - Population (GPOP), quarterly.
   - Gross Domestic Product: Implicit Price Deflator (GDPD), index, 92=100.
   - Consumer Price Index, Urban Area, All Items, $82 - 84 = 100$, monthly. The quarterly data is calculated from the average of three months.

2. Database, the Federal Reserve Bank of St. Louis

   - Civilian Employment (16 years and older), monthly.
   - Civilian participation rate, monthly.
   - Unemployment rate, monthly.

3. National Income and Products Accounts (NIPA), University of Virginia

   - Real gross domestic product, in 1992 dollar, quarterly.
   - Nominal gross domestic product, quarterly.
   - Personal expenditure on non-durable goods, nominal, quarterly.
   - Personal expenditure on service, nominal, quarterly.
   - Government consumption, nominal, quarterly.
   - Real inventory of farm industry, in 1992 dollar, quarterly.
   - Real inventory of non-durable goods, non-farm industry, in 1992 dollar, quarterly.
   - Real final sales of domestic business, in 1992 dollar, quarterly.

B. Identification of Parameters

We first list the steady state equations of the model. Mark steady state values with an asterisk. Setting the shocks to zero and requiring all real variables to be stationary, we obtain the following equations from (2.6) – (2.8) and (3.2) – (3.6):

$$c^* = bG_b(e^*, k^*)e^*q^*/(1-d); \tag{B.1}$$

$$\Delta^* = [U'(c^*) - (1-\delta_k)\omega_k^*]q^* = (1-\delta_k)\omega_k^* q^* \frac{\lambda^*}{\omega_m^*}; \tag{B.2}$$

$$\omega_m^* = bq^* \left[\theta U'(c^*) + (1-\theta)(1-\delta_k)\omega_k^*\right]; \tag{B.3}$$

$$\delta_w l^* = v^* \mu^*; \tag{B.4}$$

$$\delta_k k^* = (1-\delta_k)\left[(1-I)f(l^*) - \frac{1}{1-d}G_s(e^*, k^*)s^*q^*\right]; \tag{B.5}$$

$$U'(c^*) = (1-\delta_k)\omega_k^* \left(1 + \frac{\lambda^*}{\omega_m^*}\right); \tag{B.6}$$

$$\omega_l^* = h(v^*) \equiv H'(v^*)/\mu^*; \tag{B.7}$$

$$\frac{\gamma^*}{\beta} - 1 = G_b(e^*, k^*)e^* \frac{\lambda^*}{\omega_m^*}; \tag{B.8}$$

$$\frac{1}{\beta(1-\delta_k)} - 1 = \theta G_s(e^*, k^*)s'(k^*)q^* \frac{\lambda^*}{\omega_m^*}; \tag{B.9}$$

$$[1-\beta(1-\delta_w)]\omega_l^* = \beta\left[(1-\delta_k)\omega_k^* f'(l^*) - \varphi\right]; \tag{B.10}$$

$$\frac{\Phi'(e^*)}{G_b(e^*, k^*)} = (1-\theta)\Delta^*. \tag{B.11}$$

Next, we solve the parameters using the restrictions (i) – (v) in Section 4, together with the values of $(\beta, \gamma^*, \eta, \psi, \delta_w, \theta, \xi, \varepsilon_e, B)$. This is done for the longer period 1959:II –1998:III, but a similar exercise can be conducted for the shorter period 1959:II – 1988:I. Start with Restriction (i). Since the size of the labor force is $n_p(1+l^*)+u$ and the level of unemployment is u, Restriction (i) implies $u = 0.0611 \times 0.6148$ and $n_p(1+l^*) = 0.6148 \times (1-0.0611)$. Since the ratio of inventory to output is k^*/f^* and the ratio of inventory investment to output is $\delta_k k^*/f^*$, Restriction (ii) solves $\delta_k = 0.0065/0.9$ and leaves an equation $k^*/f^* = 0.9$ to be utilized later. Computing the income velocity of money, we can write Restriction (iii) as

$$n_p f^* \frac{p}{m} = \frac{n_p f^*}{bq^*} = \frac{f^*}{Bq^*} = 1.6882. \tag{B.12}$$

To use this equation, we divide (B.9) by (B.8) and substitute (G_b^*, G_s^*) by (2.3). Then,

$$\frac{z^* s'(k^*)q^*}{e^*} = \zeta \equiv \frac{[\beta(1-\delta_k)]^{-1} - 1}{\theta(\gamma^*/\beta - 1)}. \tag{B.13}$$

Substituting $z^* = be^*/(n_p s(k^*))$ and using (B.12), we have $\zeta \times 0.9 \times 1.6882 = k^* s'(k^*)/s^*$. Under the functional form of s, we have $k^{*(1-\varepsilon_k)} = (1-\varepsilon_k)s^* + 1$, and so $k^* s'(k^*)/s^* = 1 - \varepsilon_k + 1/s^*$. Thus,

$$s^* = (\zeta \times 0.9 \times 1.6882 - 1 + \varepsilon_k)^{-1}. \tag{B.14}$$

As ζ is known by now, this yields s^*, which implies $k^* = [(1-\varepsilon_k)s^* + 1]^{1/(1-\varepsilon_k)}$.

Also, using (B.5), we can solve for $G_s^* s^* q^*$ as

$$G_s^* s^* q^* = (1-d)\left[(1-I)f^* - \frac{\delta_k}{1-\delta_k}k^*\right] = (1-d)\left(1 - I - \frac{0.9\delta_k}{1-\delta_k}\right)f^*.$$

Since q^*, f^* and s^* are known now, this equation solves for G_s^*. Substituting this solution for G_s^* in (B.9), we can solve for λ^*/ω_m^*.

Restriction (iv) helps identify α, n_p and l^*. To see this, calculate the wage/output ratio and the hiring cost/wage ratio. Using (2.6) and $p^* = 1/(bq^*)$, Restriction (iv) implies:

$$\frac{w^* l^*}{p^* f^*} = \frac{bq^* \varphi l^*}{\omega_m^* f^*} = 0.64 \Longrightarrow \varphi = 0.64 \times \frac{\omega_m^* f^*}{bq^* l^*}; \tag{B.15}$$

$$\frac{H_0 v^{*2}}{\omega_m^* w^* l^*} = \frac{H_0 v^{*2}}{0.64 \times \omega_m^* f^* p^*} = 0.02 \Longrightarrow H_0 = 0.0128 \times \frac{\omega_m^* f^*}{bq^* v^{*2}}. \tag{B.16}$$

Substituting (B.16) and (B.4), (B.7) becomes:

$$\omega_l^* = \frac{2H_0 v^*}{\mu^*} = \frac{0.0256 \times \omega_m^* f^*}{\delta_w l^* bq^*}. \tag{B.17}$$

Using (B.3) and (B.6), we have

$$(1-\delta_k)\omega_k^* = \frac{\omega_m^*}{bq^*} \bigg/ \left(1 + \theta\frac{\lambda^*}{\omega_m^*}\right).$$

Since $f(l^*) = A^*(l^*)^\alpha$ and $A^* = 1$,

$$(1-\delta_k)\omega_k^* f'(l^*) = \frac{\omega_m^*}{bq^*} \times \frac{\alpha f^*/l^*}{1 + \theta\frac{\lambda^*}{\omega_m^*}}. \tag{B.18}$$

Substituting (B.15), (B.17) and (B.18) into (B.10), we obtain:

$$\alpha = 0.64\left(1 + \theta\frac{\lambda^*}{\omega_m^*}\right)\left\{[1 - \beta(1-\delta_w)]\frac{0.04}{\beta\delta_w} + 1\right\}. \tag{B.19}$$

Because λ^*/ω_m^* has been solved already, this equation identifies α. Then we can solve for l^* using $f^* = A^*(l^*)^\alpha$. With the earlier restriction, $n_p(1+l^*) = 0.6148(1-0.0611)$, we can identify n_p.

Restriction (v) helps identify g_0, z^* and e^*. It implies

$$e^* = 0.1117 \times 0.3 \times n_p(1+l^*)/b = 0.03351 \times (1+l^*)/B,$$

where l^* is calculated above and $B = 0.5$. We can calculate $z^* = be^*/[n_p s(k^*)]$, $g_0 = G_s^*/z^{*\xi}$, and

$$c^* = \frac{1}{1-d}G_b^* be^* q^*.$$

Then, we can calculate ω_k^* from (B.6) and ω_m^* from (B.3). Since λ^*/ω_m^* is now known, we can retrieve λ^*. Also we can pin down φ, H_0 and ω_l^* from (B.15)-(B.17) and φ_0 from (B.11). Table 2 summarizes the identified parameters and steady-state values of variables.

Table 1. Estimated VARs of money growth and log productivity

		coefficients on				covariance		test statistics*			likelihood ratio test♮	
	depend. variables	constant	γ_{t-1}	lnA_{t-1}	R^2	σ	$\rho(\gamma, lnA)$	$SC(1)$	$SC(2)$	$SC(3)$	1 vs. 2	2 vs. 3
sample	γ_t	0.840	-0.011	-0.174	0.051	0.00858	-0.177	-1061.4	-1052.8	-1047.7	5.674	12.698
1959:II		(0.000)	(0.892)	(0.005)							(0.225)	(0.013)
–1998:III	lnA_t	-0.361	0.165	0.818	0.650	0.01120	–					
		(0.000)	(0.010)	(0.000)								
sample	γ_t	0.810	-0.007	-0.188	0.075	0.00920	-0.224	-744.60	-737.44	-731.51	7.3634	9.834
1959:II		(0.000)	(0.940)	(0.004)							(0.118)	(0.043)
–1988:I	lnA_t	-0.441	0.239	0.827	0.660	0.01344	–					
		(0.000)	(0.004)	(0.000)								

* $SC(j)$ is the value of the Schwarz (1978) criterion that Hodrick et al. (1991, Table 2) used. The appropriate lag length is the one that generates the minimum of $SC(j)$.

♮ The likelihood ratio tests lag length j vs. length $j+1$. Numbers in brackets are p-values.

Table 2: Parameter Values, Shocks, and the Steady State

Parameter Values							
A^*	1	I	0.269	d	0.25	ψ	0.6
δ_w	0.06	η	4	ξ	0.8	θ	0.2
ε_e	2	ε_k	13	B	0.5		
β	0.9952	γ^*	1.01448	g_0	6.34	u	0.0380
	(0.9958)$^\natural$		(1.01727)		(6.03)		(0.0376)
n_p	0.2496	b	0.1248	δ_k	0.0072	φ	231.03
	(0.2417)		(0.1209)		(0.0072)		(248.86)
φ_0	0.5418	H_0	2151.4	α	0.6704		
	(0.5952)		(2287.8)		(0.6705)		

Steady State							
c^*	0.2227	q^*	1.4125	e^*	0.1585	v^*	0.0541
	(0.2182)		(1.4758)		(0.1601)		(0.0550)
k^*	1.1085	l^*	1.3645	ω_m^*	70.500	λ^*	1.4420
	(1.1214)		(1.3882)		(77.295)		(1.8164)
ω_k^*	401.20	ω_l^*	154.02				
	(434.32)		(165.90)				

$^\natural$The numbers in parentheses are for the period 1959:II – 1988:I.
The ones without parentheses are for the period 1959:II – 1998:III.

Table 3.1**: Simulated Moments vs. Sample Values for 1959:II - 1988:I**

	simulated moments					data
η	0.2	2	4	6	8	
$E(V_c)$	1.2237	1.2236	1.2239	1.2238	1.2235	1.2122
	(0.0035)	(0.0025)	(0.0049)	(0.0067)	(0.0072)	
$\sigma(V_c)$	0.0134	0.0138	0.0218	0.0270	0.0304	0.0239
	(0.0015)	(0.0011)	(0.0024)	(0.0033)	(0.0039)	
$cv(V_c)$	1.0924	1.1295	1.7786	2.2062	2.4853	1.9719
	(0.1210)	(0.0904)	(0.1916)	(0.2679)	(0.3212)	
$cv(y)$	1.461	1.195	1.129	1.096	1.096	1.890
$corr(V_c, g_c)$	0.1949	-0.1148	-0.1855	-0.2070	-0.2162	-0.3537
	(0.0682)	(0.0690)	(0.0504)	(0.0457)	(0.0434)	
$corr(V_c, i_{+1})$	-0.1913	0.2263	0.3354	0.3593	0.3802	0.5161
	(0.0878)	(0.0925)	(0.0814)	(0.0745)	(0.0750)	
$corr(\pi, i)$	0.9850	0.5492	0.2538	0.1134	0.0244	0.5135
	(0.0027)	(0.0615)	(0.0830)	(0.0913)	(0.0921)	
$corr(\pi, r)$	-0.3271	-0.6684	-0.7760	-0.8185	-0.8412	-0.4940
	(0.0788)	(0.0488)	(0.0347)	(0.0282)	(0.0257)	

Numbers in parentheses are standard deviations over 1000 simulations.

Table 3.2**: Simulated Moments vs. Sample Values for 1959:II - 1998:III**

	simulated moments					data
η	0.2	2	4	6	8	
$E(V_c)$	1.2640	1.2638	1.2638	1.2638	1.2639	1.2702
	(0.0028)	(0.0020)	(0.0040)	(0.0052)	(0.0061)	
$\sigma(V_c)$	0.0128	0.0131	0.0200	0.0259	0.0296	0.0224
	(0.0011)	(0.0009)	(0.0020)	(0.0027)	(0.0033)	
$cv(V_c)$	1.0112	1.0367	1.5836	2.0528	2.3379	1.7632
	(0.0882)	(0.0713)	(0.1578)	(0.2160)	(0.2599)	
$cv(y)$	1.237	1.009	0.943	0.912	0.912	1.619
$corr(V_c, g_c)$	0.1773	-0.1166	-0.1885	-0.2169	-0.2267	-0.2834
	(0.0622)	(0.0590)	(0.0466)	(0.0375)	(0.0382)	
$corr(V_c, i_{+1})$	-0.1485	0.1880	0.2801	0.3062	0.3211	0.5046
	(0.0762)	(0.0815)	(0.0738)	(0.0695)	(0.0650)	
$corr(\pi, i)$	0.9876	0.5959	0.3401	0.1658	0.0817	0.5074
	(0.0020)	(0.0499)	(0.0668)	(0.0768)	(0.0808)	
$corr(\pi, r)$	-0.2849	-0.6545	-0.7567	-0.8267	-0.8504	-0.4354
	(0.0698)	(0.0431)	(0.0333)	(0.0237)	(0.0208)	

Numbers in parentheses are standard deviations over 1000 simulations.

Table 4: Cross correlations of search intensity and inventory with productivity and money growth for 1959:II – 1998:III

	correlation of intensity e		correlation of inventory k			
	with $x = ln(A)$	with $x = \gamma$	with $x = ln(A)$	data	with $x = \gamma$	data
x_{-4}	-0.6007	0.0269	0.7780	0.7256	-0.0590	-0.1111
x_{-3}	-0.7155	0.0496	0.8702	0.7639	-0.0937	-0.1761
x_{-2}	-0.8143	0.0798	0.9091	0.7278	-0.1378	-0.2189
x_{-1}	-0.8747	0.1231	0.8537	0.6090	-0.1918	-0.2328
x	-0.8296	0.4913	0.6545	0.4122	-0.1841	-0.1791
x_{+1}	-0.5987	0.1730	0.4941	0.1318	-0.1429	-0.0698
x_{+2}	-0.4530	0.1279	0.3689	-0.0698	-0.1099	-0.0301
x_{+3}	-0.3388	0.0988	0.2715	-0.2170	-0.0838	-0.0397
x_{+4}	-0.2503	0.0759	0.1959	-0.3230	-0.0627	-0.0358

e: buyer's search intensity; k: inventory; A: productivity; γ: money growth.

Table 5: **Simulation with Restricted Model Specifications**
1959:II – 1998:III

	simulated moments without the productivity shock					data
η	0.2	2	4	6	8	
$cv(V_c)$	0.7500	0.8071	0.8553	0.8909	0.9206	1.7632
$cv(y)$	0.228	0.195	0.195	0.163	0.163	1.619
$corr(V_c, g_c)$	-0.1550	-0.1508	-0.1687	-0.1859	-0.1901	-0.2834
$corr(V_c, i_{+1})$	-0.0403	0.0382	0.0807	0.0969	0.1151	0.5046
$corr(\pi, i)$	0.9996	0.9763	0.9592	0.9293	0.9142	0.5074
$corr(\pi, r)$	-0.2879	-0.2807	-0.3236	-0.3577	-0.3751	-0.4354
	simulated moments without the money growth shock					
$cv(V_c)$	0.6645	0.6585	1.4071	1.8622	2.1478	1.7632
$cv(y)$	1.205	0.977	0.912	0.879	0.879	1.619
$corr(V_c, g_c)$	0.3659	-0.1439	-0.2112	-0.2240	-0.2306	-0.2834
$corr(V_c, i_{+1})$	-0.8778	0.8209	0.7093	0.6598	0.6331	0.5046
$corr(\pi, i)$	0.8470	-0.5440	-0.7451	-0.7992	-0.8240	0.5074
$corr(\pi, r)$	-0.6873	-0.9662	-0.9826	-0.9869	-0.9889	-0.4354
	simulated moments without labor market search					
$cv(V_c)$	1.5898	0.9589	1.1989	1.3040	1.3674	1.7632
$cv(y)$	2.260	0.852	0.623	0.524	0.492	1.619
$corr(V_c, g_c)$	0.2788	-0.1043	-0.1630	-0.1780	-0.1819	-0.2834
$corr(V_c, i_{+1})$	-0.1689	0.1569	0.2212	0.2363	0.2413	0.5046
$corr(\pi, i)$	0.9537	0.7003	0.6404	0.6156	0.6025	0.5074
$corr(\pi, r)$	-0.4239	-0.5705	-0.5925	-0.6010	-0.6030	-0.4354

Standard deviations of the statistics are roughly the same as those in Table 3.2.
The simulation generates $E(V_c) \approx 1.264$ in all three cases and for all values of η.

Table 6: Sensitivity to B, θ,ε_e, ξ and ε_k
for 1959:II – 1998:III

B	0.35	0.40	0.50*	0.60	0.65	data
$cv(V_c)$	1.5755	1.5748	1.5836	1.5796	1.5825	1.7632
	(0.1558)	(0.1522)	(0.1578)	(0.1535)	(0.1586)	
θ	0.1	0.2*	0.3	0.4	0.5	
$cv(V_c)$	1.5198	1.5836	1.6243	1.6244	1.6405	1.7632
	(0.1452)	(0.1578)	(0.1652)	(0.1617)	(0.1612)	
ε_e	1	1.5	2*	2.5	3	
$cv(V_c)$	1.4772	1.5464	1.5836	1.5886	1.6078	1.7632
	(0.1432)	(0.1477)	(0.1578)	(0.1505)	(0.1546)	
ξ	0.6	0.7	0.8*	0.85	0.9	
$cv(V_c)$	1.2652	1.4323	1.5836	1.6472	1.7139	1.7632
	(0.1024)	(0.1278)	(0.1578)	(0.1682)	(0.1800)	
ε_k	0.5	2	6	13*	16	
$cv(V_c)$	1.0730	1.1614	1.3582	1.5836	1.6449	1.7632
$cv(y)$	0.996	0.966	0.939	0.943	0.944	1.619
$corr(V_c, g_c)$	-0.1251	-0.1403	-0.1676	-0.1885	-0.1982	-0.2834
$corr(V_c, i_{+1})$	0.2479	0.2544	0.2673	0.2801	0.2820	0.5046
$corr(\pi, i)$	0.5693	0.5218	0.4329	0.3401	0.3153	0.5074
$corr(\pi, r)$	-0.5063	-0.5674	-0.6709	-0.7567	-0.7799	-0.4354
$\frac{var(output)}{var(sales)}$	6.494	5.807	4.663	3.822	3.558	

*Marked parameter values are the ones used in the benchmark.
Numbers in parentheses are standard deviations over simulations.
All simulations produce $E(V_c) \approx 1.264$.

Table 7: Simulation Results under Nash Bargaining for 1959:II – 1998:III

	Nash bargaining		benchmark model	data
	$\eta = 0.1$ $d = 0.25$	$\eta = 4$ $d = 0.87$	$\eta = 4$ $d = 0.25$	
$E(V_c)$	1.2642	1.2645	1.2638	1.2702
$cv(V_c)$	1.1127	2.5725	1.5836	1.7632
$cv(y)$	2.710	0.770	0.943	1.619
$corr(V_c, g_c)$	0.1988	-0.3511	-0.1885	-0.2834
$corr(V_c, i_{+1})$	-0.1632	0.2622	0.2801	0.5046
$corr(\pi, i)$	0.9960	-0.0102	0.3401	0.5074
$corr(\pi, r)$	-0.2239	-0.9647	-0.7567	-0.4354

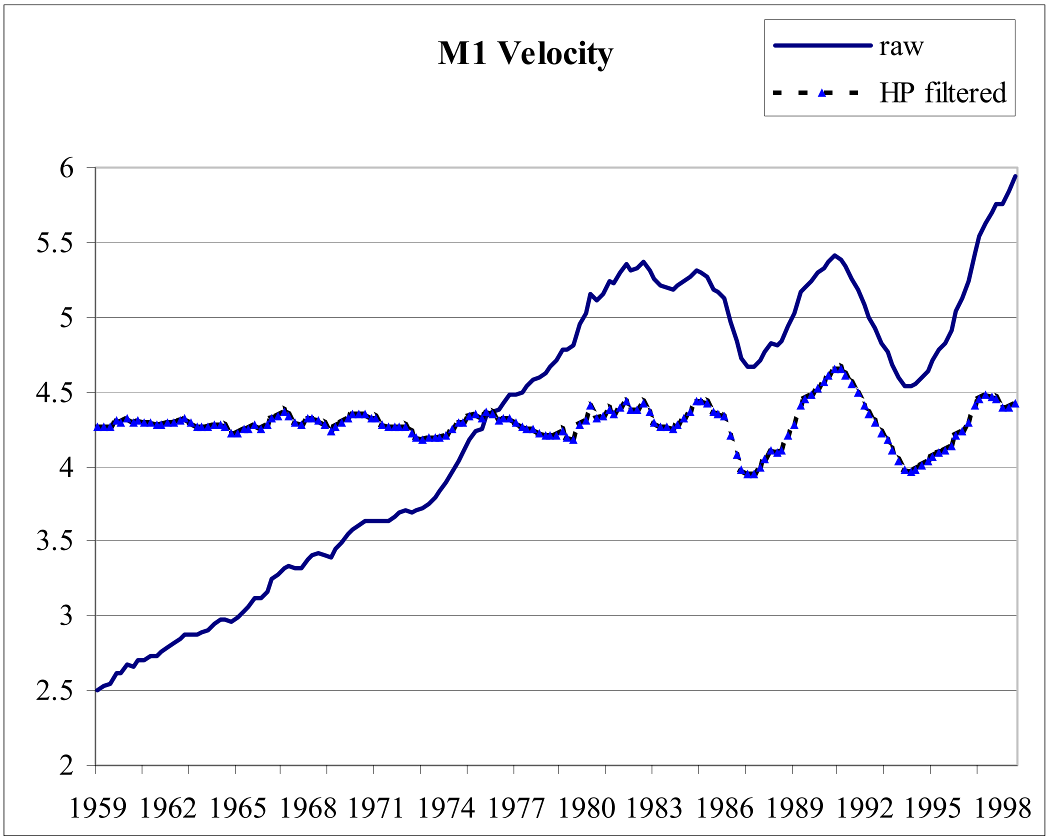

Figure 1 Velocity of $M1$ monetary aggregate

The filtering uses the Hodrick-Prescott (1980) filter.
The filtered series is plotted around the sample mean.

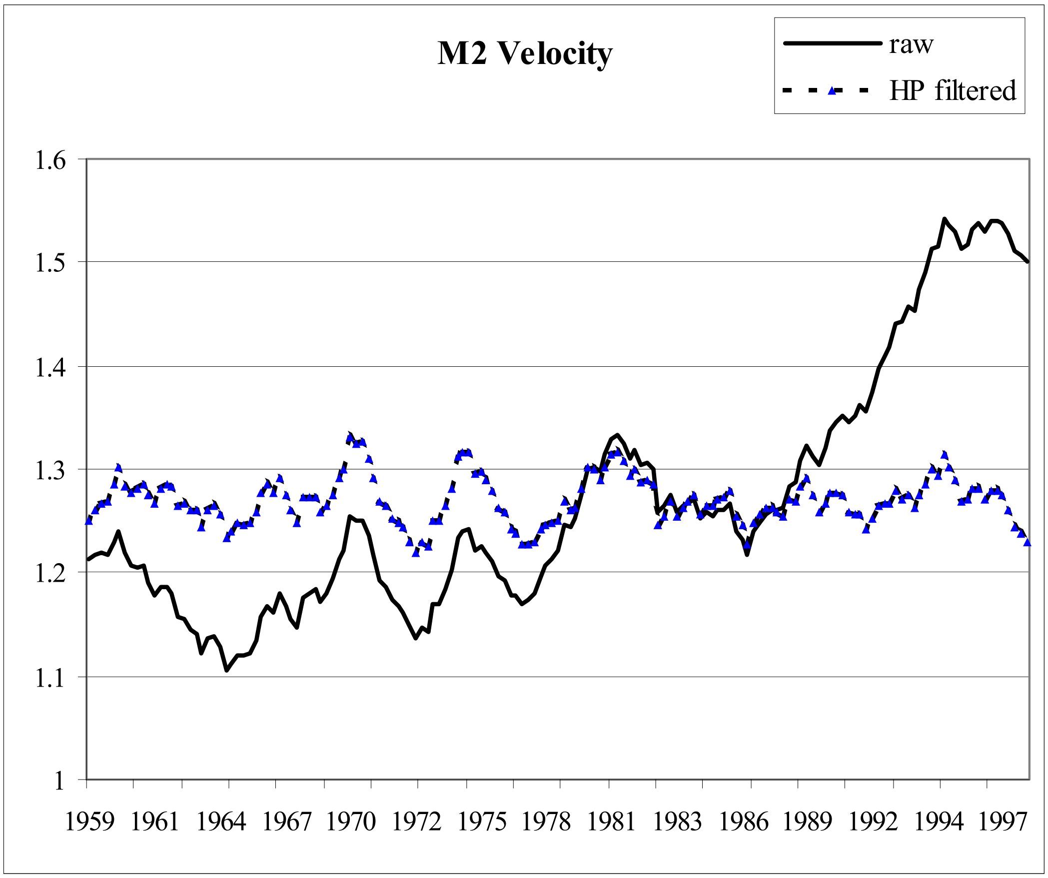

Figure 2 Velocity of $M2$ monetary aggregate

The filtering uses the Hodrick-Prescott (1980) filter.
The filtered series is plotted around the sample mean.

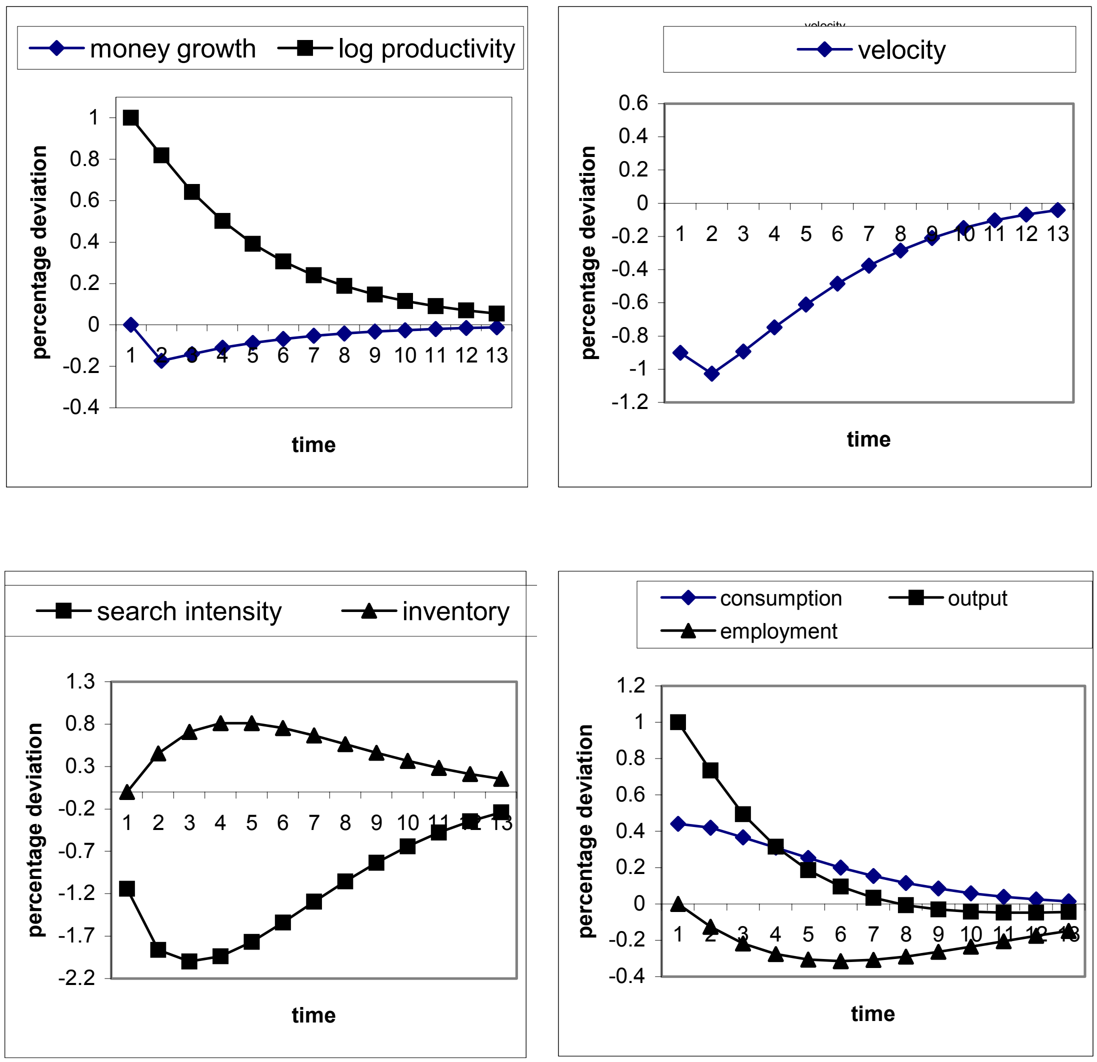

Figure 3 Impulse responses to a positive productivity shock

Notes: The economy is in the steady state at time 0 and the shock occurs at time 1. Percentage deviations from the steady state are depicted.

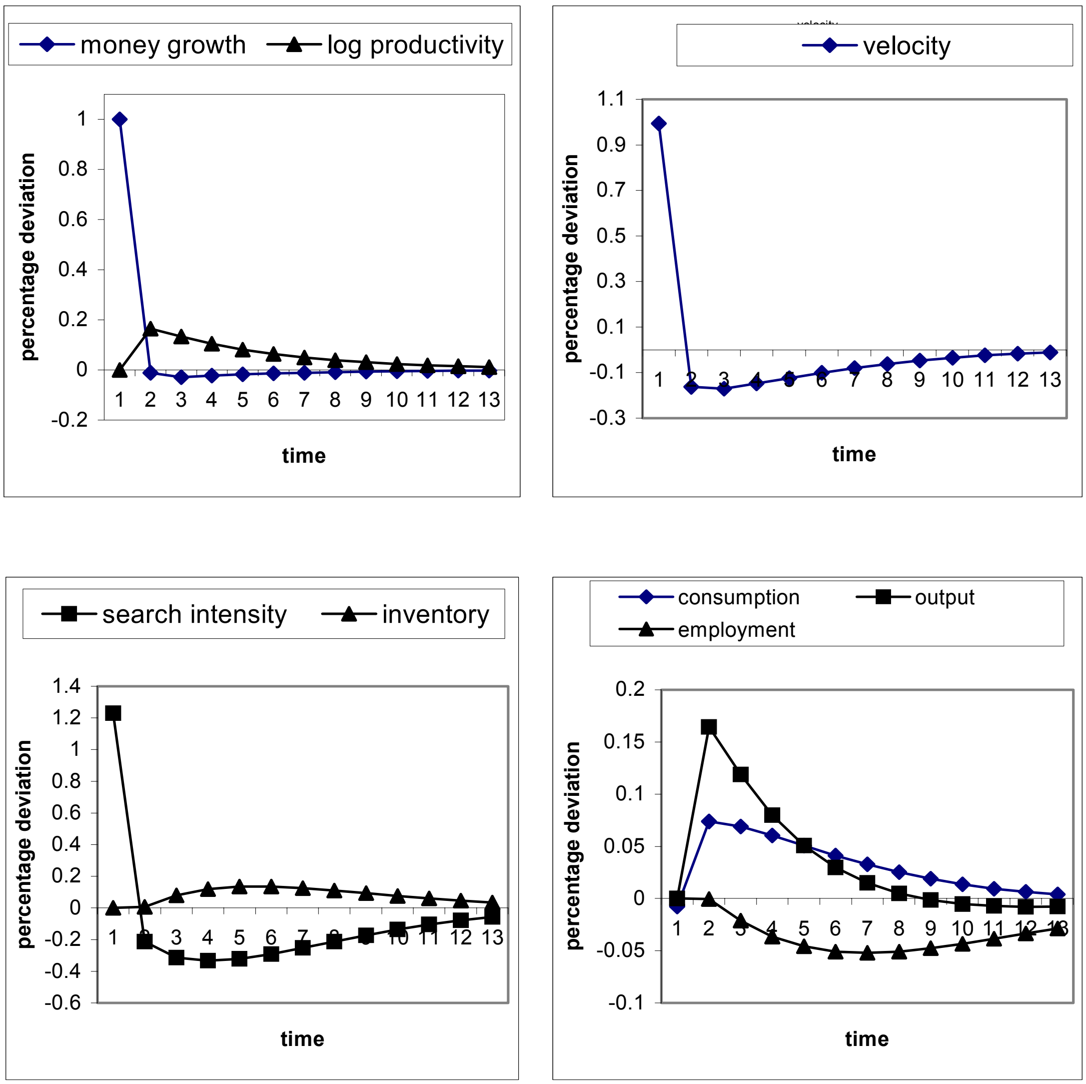

Figure 4 Impulse responses to a positive money growth shock

Notes: The economy is in the steady state at time 0 and the shock occurs at time 1. Percentage deviations from the steady state are depicted.

Supplementary Appendix

C. The Solution Method

The solution method is similar to Blanchard and Khan (1980). Below we describe the method. Notice that some of the symbols used here do not represent the same meanings as the ones in the text.

The dynamic system has exogenous state variables $Y_z \equiv (\gamma, A)^T$, two endogenous variables $Y_s \equiv (k, l)^T$, and three jump variables $Y_d \equiv (\omega_k, v_t, \omega_m)^T$. All other variables can be expressed as deterministic functions of these variables, as discussed in Section 3. The exogenous state variables are characterized by (4.1) and (4.1), while the dynamics of the other five endogenous variables are described by (3.6). Stack Y_d, Y_s and Y_z and denote the resulted 7×1 vector by Y. Then the dynamic system can be written in the following form:

$$F(Y_t, Y_{t+1}) = 0.$$

The steady state of this system Y^* such that $F(Y^*, Y^*) = 0$. Log-linearize the dynamic system, we have:

$$D \begin{bmatrix} y_t \\ \\ y_{t+1} \end{bmatrix} = 0,$$

where the ith element of y_t is $y_{it} \equiv (Y_{it} - Y_i^*)/Y_i^*$, the percentage deviation of the variable Y_{it} from its steady state ($i = 1, 2, ..., 7$). Define the vectors y_s, y_d and y_z similarly. By definition, the steady state value of y is $y^* = 0$.

To solve the saddle path of this linearized system, rewrite it as follows:

$$\begin{bmatrix} y_{st+1} \\ \\ y_{dt+1} \end{bmatrix} = W \begin{bmatrix} y_{st} \\ \\ y_{dt} \end{bmatrix} + Qy_{zt} + Ry_{zt+1}, \tag{C.1}$$

where W is a 5×5 matrix; Q and R are 5×2 matrices. For exogenous $\{y_{zt}\}$, this system has two predetermined variables and three jump variables. For the system to be saddle-path stable, the matrix W must have two stable eigenvalues (i.e., those whose absolute values are less than one) and three unstable eigenvalues (i.e., those whose absolute values are greater than one). The calibrated parameter values indeed generate such eigenvalues.

Let J_1 be a 2×2 diagonal matrix whose diagonal elements are the two stable eigenvalues, and J_2 be a 3×3 diagonal matrix whose diagonal elements are the three stable eigenvalues (the eigenvalues are ordered in increasing absolute values along the diagonals of J_1 and J_2). Denote $J = diag(J_1, J_2)$. Write W as $W = C^{-1}JC$, where C^{-1} is the eigenvector matrix corresponding to J. Decompose the matrices C, C^{-1}, Q and R as follows:

$$C = \begin{bmatrix} \underset{(2\times 2)}{C_{11}} & \underset{(2\times 3)}{C_{12}} \\ \\ \underset{(3\times 2)}{C_{21}} & \underset{(3\times 3)}{C_{22}} \end{bmatrix}, \quad C^{-1} = \begin{bmatrix} \underset{(2\times 2)}{B_{11}} & \underset{(2\times 3)}{B_{12}} \\ \\ \underset{(3\times 2)}{B_{21}} & \underset{(3\times 3)}{B_{22}} \end{bmatrix}$$

$$Q = \begin{bmatrix} \underset{(2\times 2)}{Q_1} \\ \\ \underset{(3\times 2)}{Q_2} \end{bmatrix}, \quad R = \begin{bmatrix} \underset{(2\times 2)}{R_1} \\ \\ \underset{(3\times 2)}{R_2} \end{bmatrix}.$$

For given y_{s0}, the saddle-path solution to (C.1) is:

$$\begin{aligned}
y_{st} &= B_{11}J_1B_{11}^{-1}y_{st-1} + Q_1y_{zt-1} + R_1E_{t-1}y_{zt} \\
&\quad -(B_{11}J_1C_{12} + B_{12}J_2C_{22})C_{22}^{-1} \\
&\quad \times \textstyle\sum_{j=0} J_2^{-j-1}\left[(C_{21}Q_1 + C_{22}Q_2)E_{t-1}y_{zt+j-1} + (C_{21}R_1 + C_{22}R_2)E_{t-1}y_{zt+j}\right],
\end{aligned} \tag{C.2}$$

$$\begin{aligned}
y_{dt} &= -C_{22}^{-1}C_{21}y_{st} - C_{22}^{-1} \\
&\quad \times \textstyle\sum_{j=0} J_2^{-j-1}\left[(C_{21}Q_1 + C_{22}Q_2)E_ty_{zt+j} + (C_{21}R_1 + C_{22}R_2)E_ty_{zt+j+1}\right].
\end{aligned} \tag{C.3}$$

The exogenous processes (4.1) and (4.1) can be written as $y_{zt+1} = \Gamma y_t + \varepsilon_t$, where ε_t is a vector of *iid* random variables. Then $E_t(y_{zt+j}) = \Gamma^j y_{zt}$ for all $j \geq 0$. Given a draw of innovations and initial values y_{s0}, one can calculate the time path of y_{st} and y_{dt} according to (C.2) and (C.3).